LAWS OF BOOLEAN LOGIC

Learning objectives

- 11.3.3.1 distinguish between laws of Boolean logic
- 11.3.3.2 simplify logical expressions using the laws of Boolean logic
- 11.3.3.3 Build truth tables AND, OR, NOT, NAND, NOR, XOR

Assessment criteria

Simplifies complex Boolean expression

NOT	AND	OR
¯	*	+
¬	⋀	⋁
	&	||

Laws of Boolean algebra

$$A \wedge 1 = A$$

$$A \vee 1 = 1$$

$$\neg A \wedge A = 0$$

$$A \vee 0 = A$$

$$A \wedge 0 = 0$$

$$\neg A \vee A = 1$$

De Morgan's Laws

$$\neg(A \wedge B) = \neg A \vee \neg B$$

$$\neg(A \vee B) = \neg A \wedge \neg B$$

A	B	$\bar{A}$	$\bar{B}$	$\overline{A.B}$	$\bar{A}+\bar{B}$
0	0				
0	1				
1	0				
1	1				

A	B	$\bar{A}$	$\bar{B}$	$\overline{A+B}$	$\bar{A}.\bar{B}$
0	0				
0	1				
1	0				
1	1				

De Morgan's Laws

$$\neg(A \wedge B) = \neg A \vee \neg B$$

$$\neg(A \vee B) = \neg A \wedge \neg B$$

A	B	$\overline{A}$	$\overline{B}$	$\overline{AB}$	$\overline{A}+\overline{B}$
0	0	1	1		
0	1	1	0		
1	0	0	1		
1	1	0	0		

A	B	$\overline{A}$	$\overline{B}$	$\overline{A+B}$	$\overline{A}.\overline{B}$
0	0	1	1		
0	1	1	0		
1	0	0	1		
1	1	0	0		

De Morgan's Laws

$$\neg(A \wedge B) = \neg A \vee \neg B$$

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A	B	$\overline{A}$	$\overline{B}$	$\overline{A.B}$	$\overline{A}+\overline{B}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

A	B	$\overline{A}$	$\overline{B}$	$\overline{A+B}$	$\overline{A}.\overline{B}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

Double complement law

$$\neg\neg A = A$$

DeMorgan's Law	$\overline{(xy)} = \overline{x}+\overline{y}$	$\overline{(x+y)} = \overline{x}\,\overline{y}$
Double Complement Law	$\overline{\overline{x}} = x$	

Commutative rule

$$A \wedge B = B \wedge A$$

$$A \vee B = B \vee A$$

In algebra:

$ab=ba$

$a+b = b+$

The Associativity Rule

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

In algebra:

$$(ab)c = a(bc)$$

$$(a+b)+c = a+(b+c)$$

Rule of distributivity

$$A \wedge (B \vee C) = A \wedge B \vee A \wedge C$$

In algebra:

$$a(b+c)=ab+ac$$

Formative assessment 1

Prove the law using the truth table

The laws of Boolean Logic

Identity Name	AND Form	OR Form
Identity Law	$1x = x$	$0+x = x$
Null (or Dominance) Law	$0x = 0$	$1+x = 1$
Idempotent Law	$xx = x$	$x+x = x$
Inverse Law	$x\bar{x} = 0$	$x+\bar{x} = 1$
Commutative Law	$xy = yx$	$x+y = y+x$
Associative Law	$(xy)z = x(yz)$	$(x+y)+z = x+(y+z)$
Distributive Law	$x+yz = (x+y)(x+z)$	$x(y+z) = xy+xz$
Absorption Law	$x(x+y) = x$	$x+xy = x$
DeMorgan's Law	$(\overline{xy}) = \bar{x}+\bar{y}$	$(\overline{x+y}) = \bar{x}\bar{y}$
Double Complement Law	$\bar{\bar{x}} = x$	

Table 30.13 Explanations of the main identities and rules

A.B = B.A	The order in which two variables are ANDed makes no difference
A+B = B+A	The order in which two variables are ORed makes no difference
A.0 = 0	A variable ANDed with 0 equals 0
A+1 = 1	A variable ORed with 1 equals 1
A+0 = A	A variable ORed with 0 equals the variable
A.1 = A	A variable ANDed with 1 equals the variable
A.A = A	A variable ANDed with itself equals the variable
A+A = A	A variable ORed with itself equals the variable
$A.\overline{A} = 0$	A variable ANDed with its inverse equals 0
$A+\overline{A} = 1$	A variable ORed with its inverse equals 1
$\overline{\overline{A}} = A$	A variable that is double inversed equals the variable
(A.B).C = A.(B.C)	It makes no difference how the variables are grouped together when ANDed
(A+B)+C = A+(B+C)	It makes no difference how the variables are grouped together when ORed
A.(B+C) = A.B+A.C	The expression can be distributed or factored out, meaning that variables can be moved in and out of brackets either side of the expression. In English this expression would be A AND (B OR C) = (A AND B) OR (A AND C).

How to simplify logic expression.

$$Q = A.B.\overline{C} + A.B.C + A.\overline{B}$$

$$Q = A.B.\overline{C} + A.B.C + A.\overline{B}$$
$$Q = A.(B.\overline{C} + B.C + \overline{B})$$
$$Q = A.(B.(\overline{C} + C)) + \overline{B})$$
$$Q = A.(B.(1) + \overline{B})$$

Using the first identity **B.1 = B** so the expression becomes

$$Q = A.(B + \overline{B})$$

Using our sixth identity again the term $B + \overline{B} = 1$ so the expression now becomes:

$$Q = A.1$$

Using the first identity **A.1 = A** so the expression finally becomes

$$Q = A$$

$$Q = B.C.(\overline{C} + D) + C.D + C + \overline{A}$$

Solution:

$$Q = B.C.(\overline{C} + D) + C.D + C + \overline{A}$$

$$Q = B.C.\overline{C} + B.C.D + C.D + C + \overline{A}$$

$$Q = B.0 + B.C.D + C.(D + 1) + \overline{A}$$

$$Q = B.C.D + C + \overline{A}$$

$$Q = C.(B.D + 1) + \overline{A}$$

$$Q = C.1 + \overline{A}$$

$$Q = C + \overline{A}$$

Formative assessment 2

Prove the law using the logic laws

Simplify the following expressions using logic laws

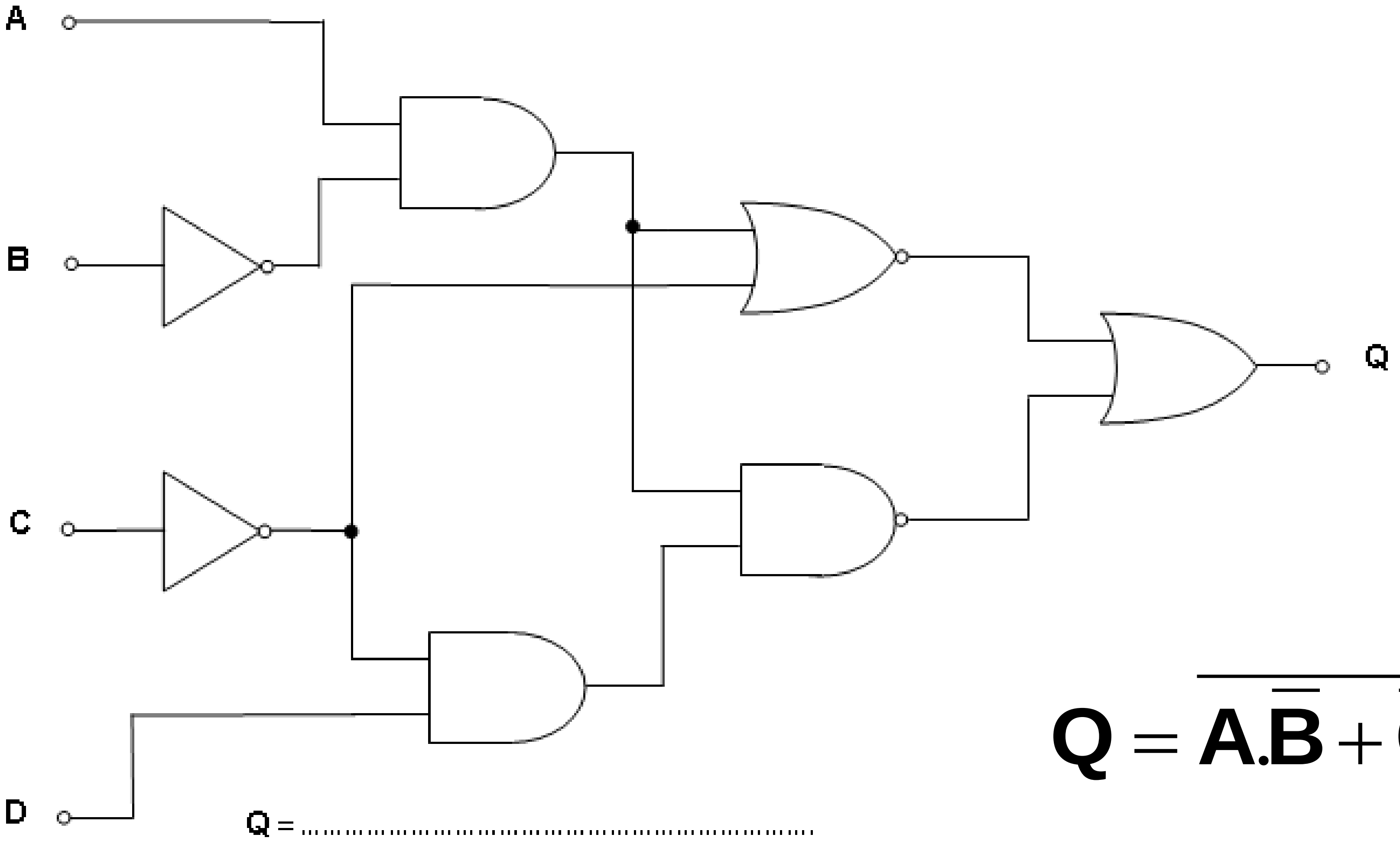
Formative assessment 2

- 1 $Q = A.B.\overline{C} + A.\overline{C} + \overline{A}.\overline{C}.D + \overline{A}.\overline{C}.\overline{D}$
- 2 $Q = A.B.(\overline{B} + C) + B.C + B$
- 3 $Q = B.(A + \overline{C}) + A + A.(\overline{A} + B)$
- 4 $Q = A.B.\overline{C} + B.C + \overline{A}.B.\overline{C} + A.B.\overline{B}$
- 5 $Q = A.B.C + B.C.D + B.C.\overline{D} + B.\overline{C}.D + A.B.\overline{C} + \overline{A}.B.\overline{C}$
- 6 $Q = A.B.C.D + A.B.D + A.\overline{B}.D + A.\overline{B}.C.D + A.C.D + \overline{A}.C.D$

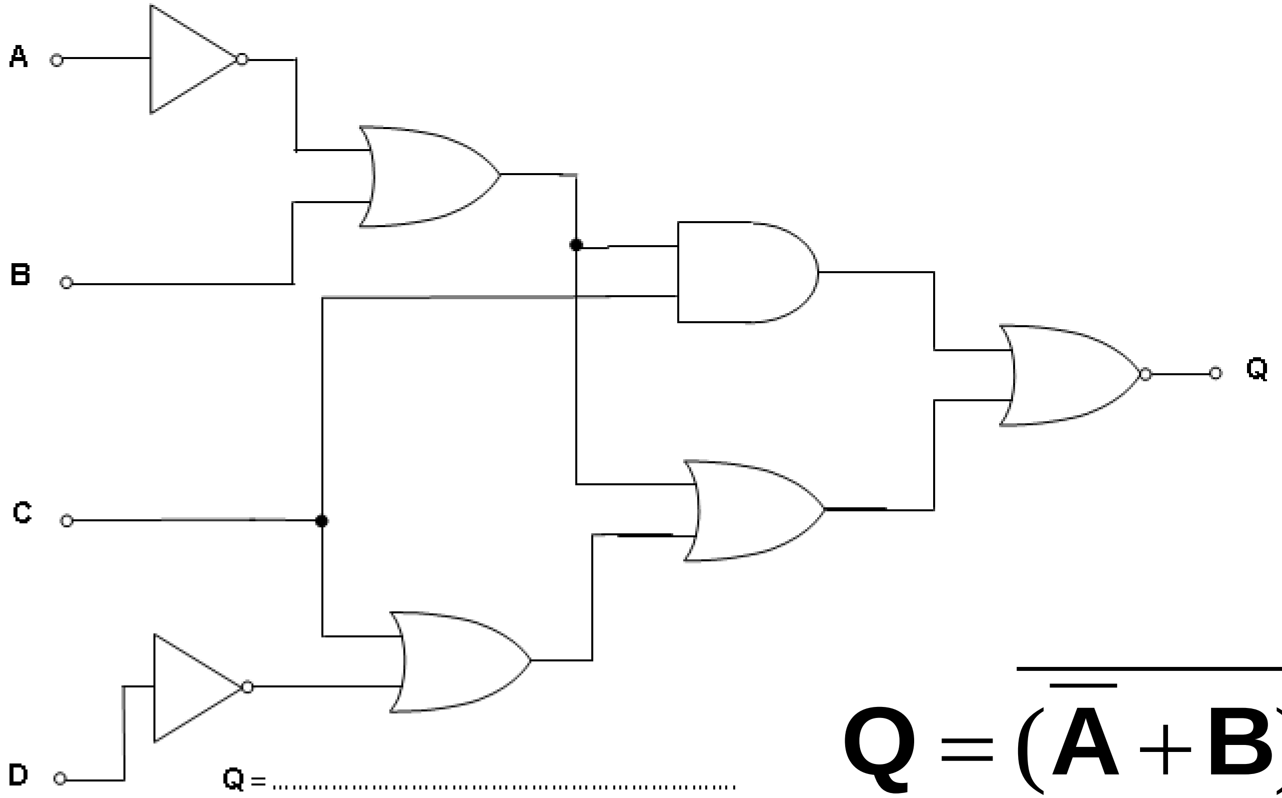
Answers

1. ¬C
2. B
3. A+B* ¬ C
4. B
5. B
6. AD?

Derive the Boolean Expression for the output of the following logic systems.



$$\mathbf{Q} = \overline{\mathbf{A}.\overline{\mathbf{B}} + \overline{\mathbf{C}}} + \overline{(\mathbf{A}.\overline{\mathbf{B}}).(\mathbf{D}.\overline{\mathbf{C}})}$$



$$Q = \overline{(\overline{A} + B).C + (\overline{A} + B) + (C + \overline{D})}$$

Simplify the formula and build logic scheme

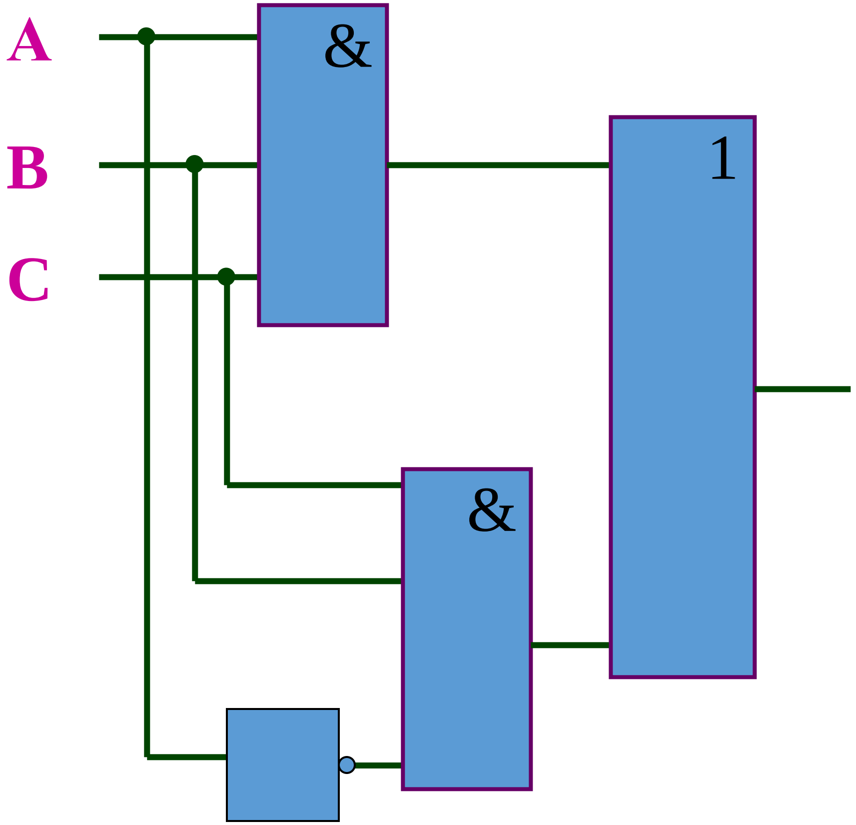
Formative assessment 3

- 1 $A \cdot B \cdot C \vee \overline{A} \cdot B \cdot C$
- 2 $(\overline{A} \vee \overline{B} \vee \overline{C}) \cdot (\overline{A} \vee B \cdot C)$
- 3 $A \cdot \overline{C} \vee C \cdot (B \vee \overline{C}) \vee (A \vee \overline{B}) \cdot C$

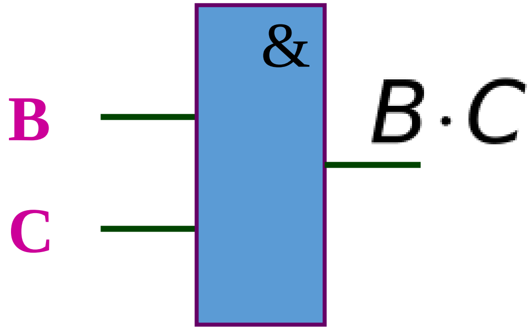
Simplify the formula

$$A \cdot B \cdot C \vee \bar{A} \cdot B \cdot C$$

SCHEME



SIMPLIFIED LOGICAL SCHEME



Answer:

$$A\cdot B\cdot C \vee \overline{A}\cdot B\cdot C =$$

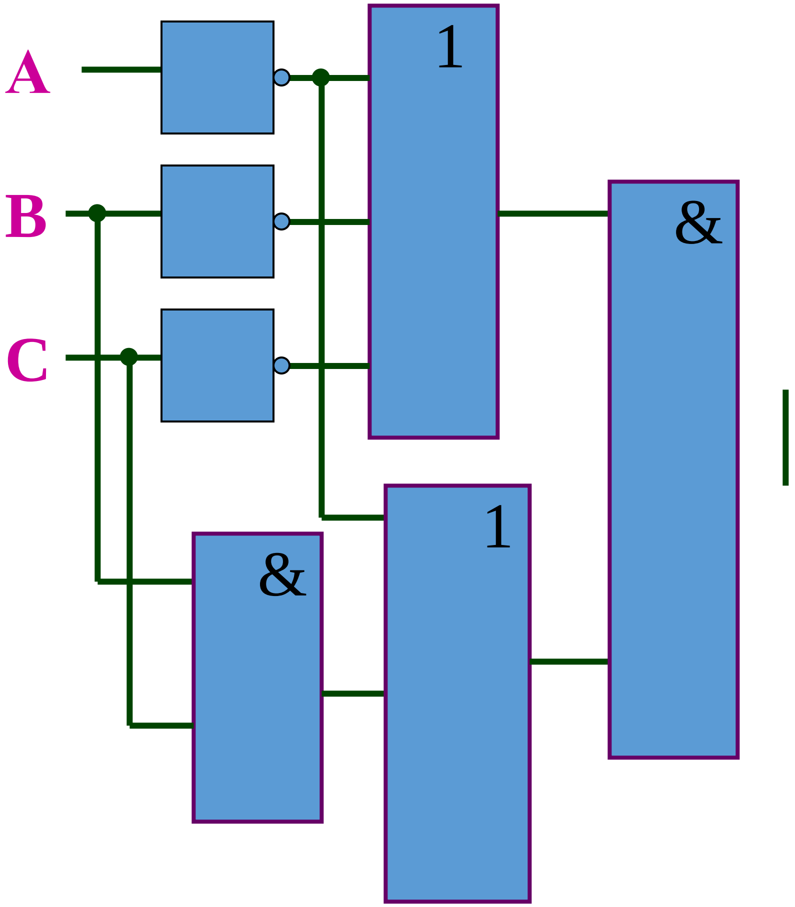
$$= B\cdot C\cdot\left(A\vee\overline{A}\right) =$$

$$= B\cdot C\cdot 1 = B\cdot C$$

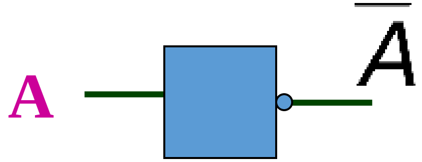
Simplify the formula

$$\left(\overline{A} \vee \overline{B} \vee \overline{C}\right) \cdot \left(\overline{A} \vee B \cdot C\right)$$

SCHEME



SIMPLIFIED LOGICAL SCHEME



DECISION:

$$(\bar{A} \vee \bar{B} \vee \bar{C}) \cdot (\bar{A} \vee B \cdot C) =$$

$$= (\bar{A} \vee \overline{B \cdot C}) \cdot (\bar{A} \vee B \cdot C) =$$

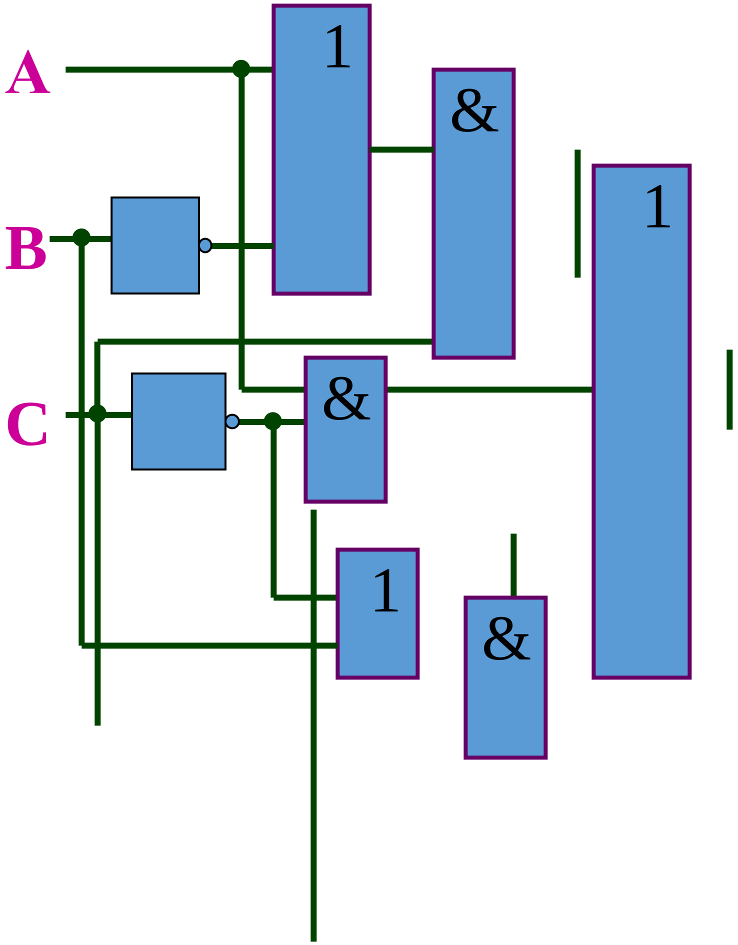
$$= \bar{A} \vee \overline{B \cdot C} \cdot B \cdot C = \bar{A} \vee 0 =$$

$$= \bar{A}$$

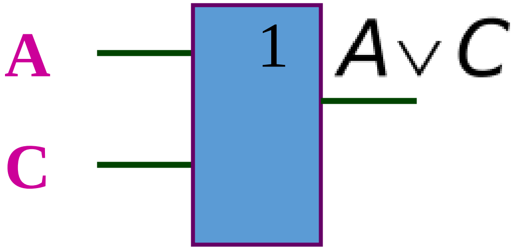
Simplify the formula

$$A \cdot \overline{C} \vee C \cdot \left(B \vee \overline{C}\right) \vee \left(A \vee \overline{B}\right) \cdot C$$

SCHEME



SIMPLIFIED LOGICAL SCHEME



РЕШЕНИЕ:

$$A\cdot\overline{C}\vee C\cdot\left(B\vee\overline{C}\right)\vee\left(A\vee\overline{B}\right)\cdot C=$$

$$=A\cdot\overline{C}\vee C\cdot B\vee C\cdot\overline{C}\vee A\cdot C\vee\overline{B}\cdot C=$$

$$=\left(A\cdot\overline{C}\vee A\cdot C\right)\vee\left(C\cdot B\vee\overline{B}\cdot C\right)\vee 0=$$

$$=A\cdot\left(\overline{C}\vee C\right)\vee C\cdot\left(B\vee\overline{B}\right)=A\vee C$$

Formative assessment 4

RESOURCES

Read:https://www.kullabs.com/classes/subjects/units/lessons/notes/note-detail/3479

Read 2: https://www.allaboutcircuits.com/technical-articles/boolean-identities/