10. (a) Write down the value of the real root of the equation

$$x^3 - 64 = 0.$$

(1)

(b) Find the complex roots of $x^3 - 64 = 0$, giving your answers in the form $a + ib$, where a and b are real.

(4)

(c) Show the three roots of $x^3 - 64 = 0$ on an Argand diagram.

(2)
(Total 7 marks)

7 (i) Find the roots of the equation

$$z^2 + (2\sqrt{3})z + 4 = 0,$$

giving your answers in the form $x + iy$, where x and y are real. [2]

(ii) State the modulus and argument of each root. [3]

(iii) Showing all your working, verify that each root also satisfies the equation

$$z^6 = -64.$$ [3]

6. Given that 2 and $5 + 2i$ are roots of the equation

$$x^3 - 12x^3 + cx + d = 0, \qquad c, d \in \mathbb{R},$$

(a) write down the other complex root of the equation.

(1)

(b) Find the value of c and the value of d.

(5)

(c) Show the three roots of this equation on a single Argand diagram.

(2)
(Total 8 marks)

2. $$f(x) = 2x^3 - 5x^2 + px - 5, p \in \mathbb{R}$$

Given that $1 - 2i$ is a complex solution of $f(x) = 0$,

(a) write down the other complex solution of $f(x) = 0$,

(1)

(b) solve the equation $f(x) = 0$,

(6)

(c) find the value of p.

(2)
(Total 9 marks)

17. Given that $3 - 2i$ is a solution of the equation

$$x^4 - 6x^3 + 19x^2 - 36x + 78 = 0,$$

(a) solve the equation completely,

(7)

(b) show on a single Argand diagram the four points that represent the roots of the equation.

(2)

(Total 9 marks)

26. (a) By factorisation, show that two of the roots of the equation $x^3 - 27 = 0$ satisfy the quadratic equation $x^2 + 3x + 9 = 0$.

(2)

(b) Hence, or otherwise, find the three cube roots of 27, giving your answers in the form $a + ib$, where $a, b \in \mathbb{R}$.

(3)

(c) Show these roots on an Argand diagram.

(2)

(Total 7 marks)

21. Given that $3 + i$ is a root of the equation $f(x) = 0$, where

$$f(x) = 2x^3 + ax^2 + bx - 10, \qquad a, b \in \mathbb{R},$$

(a) find the other two roots of the equation $f(x) = 0$,

(4)

(b) find the value of a and the value of b.

(3)

(Total 7 marks)

19. Given that $1 + 3i$ is a root of the equation $z^3 + 6z + 20 = 0$,

(a) find the other two roots of the equation,

(3)

(b) show, on a single Argand diagram, the three points representing the roots of the equation,

(1)

(c) prove that these three points are the vertices of a right-angled triangle.

(2)

(Total 6 marks)