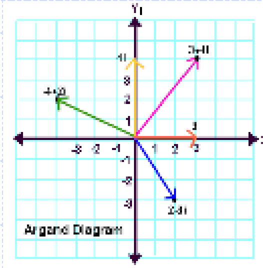
Алгебраның негізгі теоремасы, жоғары дәрежелі қарапайым теңдеулерді шешу

Оқу мақсаттары:

- 12.2.2.3 алгебраның негізгі теоремасын және оның салдарын біледі және қолданады;

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that -1 is a root of the equation:

$$x^3 - x^2 + 3x + k = 0$$

Find the other two roots of the equation.

→ If we substitute -1 in, the equation will balance…

$$x^3 - x^2 + 3x + 5 = 0$$

$$x^3 - x^2 + 3x + k = 0$$

Sub in x = -1

$$(-1)^3 - (-1)^2 + 3(-1) + k = 0$$

Calculate each part

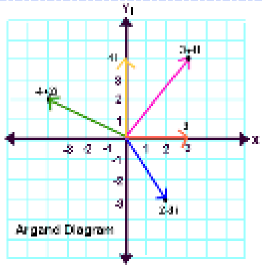
$$-1 - 1 - 3 + k = 0$$

Rearrange to fin d k

$$k = 5$$

We now know the actual equation

$$x^3 - x^2 + 3x + 5 = 0$$



Complex Numbers

You can solve some types of polynomial equation with real coefficients

Given that -1 is a root of the equation:

$$x^3 - x^2 + 3x + k = 0$$

Find the other two roots of the equation.

- We can now solve the equation

$$x^3 - x^2 + 3x + 5 = 0$$

- As -1 is a root, (x + 1) will be a factor of the equation

Divide x^3 by x

Multiply the divisor by the answer and write it beneath

Subtract this from the original equation

Now divide $-2x^2$ by x

Multiply the divisor by this and continue these steps until you're finished!

$$x^2 - 2x + 5$$

$$x \;\big)\; x^3 - x^2 + 3x$$

$$x^3 + x^2 \quad -$$

$$-2x^2 \; 3x + 5$$

$$-2x^2 \quad -$$

$$5x + 5$$

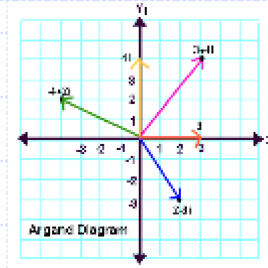
$$5x + 5 \quad -$$

$$0$$

$$x^3 - x^2 + 3x + 5$$

$$= (x + 1)(x^2 - 2x + 5)$$

Complex Numbers



You can solve some types of polynomial equation with real coefficients

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Find the other two roots of the equation.

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$$x^3 - x^2 + 3x + 5 = 0$$

- As -1 is a root, (x + 1) will be a factor of the equation

$$(x+1)(x^2 - 2x + 5) = 0$$

$$x^3 - x^2 + 3x + 5 = 0$$

$$(x+1)(x^2 - 2x + 5) = 0$$

Either this bracket is 0

Or this bracket is 0

$$x + 1 = 0$$

$$x = -1$$

We already knew this solution!

$$x^2 - 2x + 5 = 0$$

Use completing the square

$$(x-1)^2 + 4 = 0$$

Subtract 4

$$(x-1)^2 = -4$$

Square root

$$x - 1 = \pm 2i$$

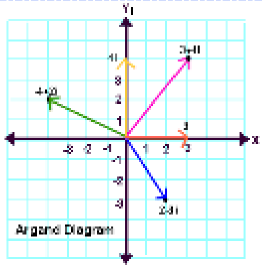
Add 1

$$x = 1 \pm 2i$$

The solutions of the equation $x^3 - x^2 + 3x + 5 = 0$ are:

$$x = -1 \quad x = 1 + 2i \quad and \quad x = 1 - 2i$$

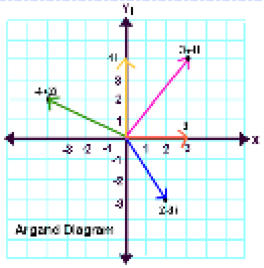
Complex Numbers



You can solve some types of polynomial equation with real coefficients

In a cubic equation, either:
→ All 3 solutions are real
→ One solution is real and the other 2 form a complex conjugate pair

Complex Numbers



You can solve some types of polynomial equation with real coefficients

Given that 3 + i is a root of the quartic equation:

$$2x^4 - 3x^3 - 39x^2 + 120x - 50 = 0$$

Solve the equation completely.

As one root is 3 + i, we know that another root will be 3 – i

→ We can use these to find an expression which will factorise into the original equation

$$x^2 - 6x + 10 \ is \ a \ factor$$

→ Divide the original equation by this!

$$(x^2 - 6x + 10)(2x^2 + 9x - 5) = 0$$

We already have the solutions for this bracket!

We need to find the solutions for this one!

$$3 + i$$
$$3 - i$$

$$2x^2 + 9x - 5 = 0$$

Factorise

$$(2x + 1)(x - 5) = 0$$

$$x = -\frac{1}{2} \quad or \quad x = 5$$

$$2x^4 - 3x^3 - 39x^2 + 120x - 50 = 0$$

$$Solutions\ are: \quad x = 3 + i$$
$$x = 3 - i$$
$$x = -\frac{1}{2}$$
$$x = 5$$

All these will give the answer 0 when substituted in!

Қосымша 1

Exam-Style Questions

Solve each of the following over the complex number field

$$z^4 = 8 - 8\sqrt{3}i$$

(level B)

10. (a) Write down the value of the real root of the equation

$$x^3 - 64 = 0.$$

(1)

(b) Find the complex roots of $x^3 - 64 = 0$, giving your answers in the form $a + ib$, where a and b are real.

(4)

(c) Show the three roots of $x^3 - 64 = 0$ on an Argand diagram.

(2)

(Total 7 marks)

(level B)

7 (i) Find the roots of the equation

$$z^2 + (2\sqrt{3})z + 4 = 0,$$

giving your answers in the form $x + iy$, where x and y are real. [2]

(ii) State the modulus and argument of each root. [3]

(iii) Showing all your working, verify that each root also satisfies the equation

$$z^6 = -64.$$ [3]

(level C)

6. Given that 2 and 5 + 2i are roots of the equation

$$x^3 - 12x^3 + cx + d = 0, \qquad c, d \in \mathbb{R},$$

(a) write down the other complex root of the equation.

(1)

(b) Find the value of c and the value of d.

(5)

(c) Show the three roots of this equation on a single Argand diagram.

(2)

(Total 8 marks)

(level C)

2. $$f(x) = 2x^3 - 5x^2 + px - 5, \; p \in \mathbb{R}$$

Given that $1 - 2i$ is a complex solution of $f(x) = 0$,

(a) write down the other complex solution of $f(x) = 0$,

(1)

(b) solve the equation $f(x) = 0$,

(6)

(c) find the value of p.

(2)

(Total 9 marks)

(level C)

17. Given that $3 - 2i$ is a solution of the equation

$$x^4 - 6x^3 + 19x^2 - 36x + 78 = 0,$$

(a) solve the equation completely,

(7)

(b) show on a single Argand diagram the four points that represent the roots of the equation.

(2)

(Total 9 marks)

26. (a) By factorisation, show that two of the roots of the equation $x^3 - 27 = 0$ satisfy the quadratic equation $x^2 + 3x + 9 = 0$.

(2)

(b) Hence, or otherwise, find the three cube roots of 27, giving your answers in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(3)

(c) Show these roots on an Argand diagram.

(2)

(Total 7 marks)

21. Given that $3 + \mathrm{i}$ is a root of the equation $\mathrm{f}(x) = 0$, where

$$\mathrm{f}(x) = 2x^3 + ax^2 + bx - 10, \qquad a, b \in \mathbb{R},$$

(a) find the other two roots of the equation $\mathrm{f}(x) = 0$,

(4)

(b) find the value of a and the value of b.

(3)

(Total 7 marks)

19. Given that $1 + 3i$ is a root of the equation $z^3 + 6z + 20 = 0$,

(a) find the other two roots of the equation,

(3)

(b) show, on a single Argand diagram, the three points representing the roots of the equation,

(1)

(c) prove that these three points are the vertices of a right-angled triangle.

(2)

(Total 6 marks)

3 things I've learned today..

2 things I found interesting..

1 question I still have..