Nazarbayev Intellectual Schools

Cambridge International Examinations
In collaboration with Nazarbayev Intellectual Schools, Kazakhstan

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

0411152895

MATHEMATICS **Grade 12**

Paper 1 **May 2014**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical Instruments
List of Formulae and Statistical Tables

12MATH/01

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do **not** use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
Calculators **not** allowed.

You may lose marks if you do not show your working or if you do not use appropriate units.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 30.

For Examiner's Use	
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Total	

This document consists of **13** printed pages and **3** blank pages.

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For Examiner's Use

1 Factorise completely.

$$x^4 - 16$$

.... [1]

2 Solve the equation $\dfrac{x}{x+3} = \dfrac{x-3}{x-6}$.

.... [1]

3 Find the remainder when

$$x^3 + 2x^2 - 3x + 4$$

is divided by $(x-2)$.

.... [1]

For Examiner's Use

4 Find the real roots of the equation $x^3 = 7 + \dfrac{8}{x^3}$.

.... [1]

5 Solve the equation $|2x-1| = |2x+3|$.

.... [1]

6 Find the equations of the asymptotes of the curve $y = \dfrac{x^2+5x+3}{x}$.

.... [1]

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7 Solve the equation $4\mathrm{e}^{x} = 9\mathrm{e}^{-x}$, giving your answer in the form $\ln k$.

.... [1]

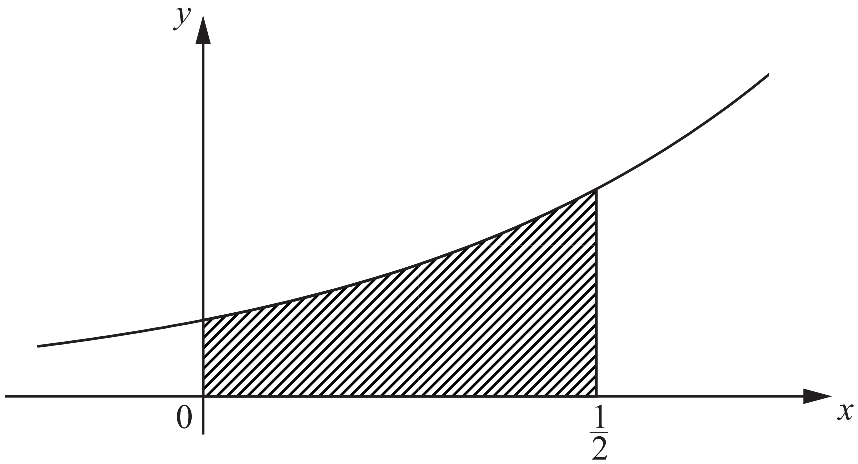
8 A curve has equation $y = 5x - 6\ln(x+1)$. Find the gradient of the curve at the point for which $x = 2$.

.... [1]

9 The equation of a curve is $y = 4x\mathrm{e}^{\frac{1}{2}x}$. Find the x-coordinate of the point at which $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

.... [1]

10



The diagram shows the curve $y = 6e^{2x}$. Find the area of the shaded region.

.... [2]

11 Find the value of $\int_0^{\frac{1}{6}\pi} 2\sin 3x \, dx$.

.... [1]

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12 Solve the equation $2x - \frac{1}{3}\pi = \arcsin\frac{1}{2}$, giving the answer in the form $k\pi$.

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.... [1]

13

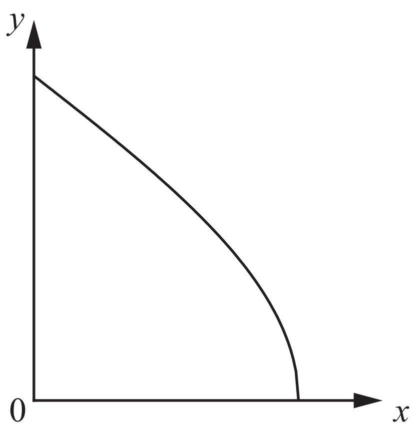

Diagram 1

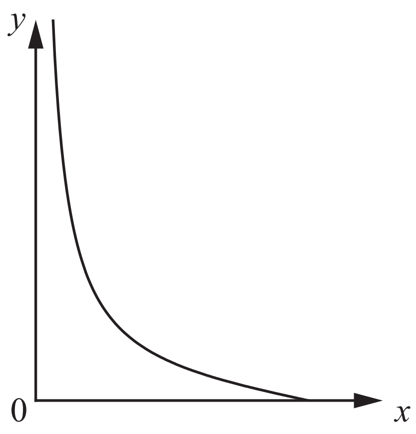

Diagram 2

Each diagram shows part of a curve, the equation of which is one of the following:

$y = \arcsin x$, $y = \arccos x$, $y = \arctan x$,

$y = \operatorname{cosec} x$, $y = \cot x$, $y = \sec x$.

State the equation of the curve in Diagram 1 and the equation of the curve in Diagram 2.

Diagram 1: ....

Diagram 2: .... [1]

For Examiner's Use

14 Simplify $\sin(x+30°)-\sin(x-30°)$.

.... [1]

15 The origin is the centre of a sphere. The point (–2, 5, 1) lies on the surface of the sphere. Find the equation of the sphere.

.... [1]

For Examiner's Use

16 A straight line has the vector equation

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + s(-\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$$

The point (4, b, c) lies on this line. Find the value of b and the value of c.

.... [1]

17 The point A is (2, –7, 1) and the point B is (8, –10, –11). Find the coordinates of the point C that divides AB in the ratio 2 : 1.

.... [1]

For Examiner's Use

18 Two planes have equations

$$2x - y + 5z = 6 \text{ and } ax + 4y - 2z = 1.$$

Given that the planes are perpendicular, find the value of the constant *a*.

.... [1]

19 The complex numbers *z* and *w* are defined by

$$z = 4 - 2\text{i} \text{ and } w = 1 + 2\text{i}.$$

Simplify $zw - 3w^*$, where w^* denotes the complex conjugate of w.

.... [1]

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20 On a single Argand diagram, shade the region representing the complex numbers satisfying both of the following loci:

$$|z+1| \le 3 \text{ and } -\frac{1}{4}\pi \le \arg z \le \frac{1}{4}\pi .$$

[2]

21 The complex number z is defined by

$$z = \cos\frac{1}{5}\pi + \mathrm{i}\sin\frac{1}{5}\pi .$$

Express the complex number z^8 in the form $\cos\theta + \mathrm{i}\sin\theta$, where $-\pi < \theta \le \pi$.

.... [1]

For Examiner's Use

22 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4x = 0.$$

.... [1]

23 The complex number z is defined by $z = 1 + i$. It is given that n is a positive integer. Given also that z^n is a real number, what can you deduce about the value of n?

.... [2]

For Examiner's Use

24 It is given that θ is the acute angle such that $\theta = \arctan\frac{1}{3}$. Find the value of $\sin 2\theta$.

.... [2]

For Examiner's Use

25 **M** is the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and **I** is the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The determinant of **M** is equal to k.

Given that the determinant of the matrix $\mathbf{M} + \mathbf{I}$ is equal to $k + 1$, show that $a + d = 0$.

[2]

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