Cambridge International Examinations
In collaboration with Nazarbayev Intellectual Schools, Kazakhstan

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

3082957701

MATHEMATICS **Grade 12**

Paper 2 **May 2014**

2 hours

Candidates answer on the Question Paper.

Additional Materials: Calculator
Geometrical Instruments
List of Formulae and Statistical Tables **12MATH/02**

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do **not** use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
Electronic calculators may be used.

You may lose marks if you do not show your working or if you do not use appropriate units.

Give non-exact numerical answers correct to 3 significant figures and angles correct to 0.1° unless a different degree of accuracy is specified in the question.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
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This document consists of **19** printed pages and **1** blank page.

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1 Use the laws of logarithms to solve the equation

$$1 + \log_3 (1 + x^2) = \log_3(10x).$$

For Examiner's Use

..... [4]

2 **(a)** Expand $(1 - 3x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 .

For Examiner's Use

.....

[3]

(b) Hence show that $\sqrt{0.4} \approx 0.64$.

[2]

3 **(a)** Find the exact value of $\cos x$ for which $2\cos x + \cos 2x = 2$.

For Examiner's Use

..... [3]

(b) Find the values of x for which $2\cos x + \cos 2x = 2$, for $0° \leq x \leq 360°$, giving your values correct to the nearest $0.1°$.

..... [2]

4 Given that $\mathbf{A} = \begin{pmatrix} a & 1 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}$, find the matrix **M** such that **MB = A**.

For Examiner's Use

..... [5]

5 The arithmetic mean of six numbers is 9. The variance of the numbers is 2. When an extra number is included, the arithmetic mean of the seven numbers is 8.5. Find

For Examiner's Use

(a) the extra number,

..... [2]

(b) the variance of the seven numbers.

..... [4]

6 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x\mathrm{e}^{2x}}{\mathrm{e}^{y}}.$$

It is given that $y = 1$ when $x = 0$.

Find an expression for y in terms of x.

For Examiner's Use

..... [7]

7 The quadratic equation $2x^2 + 7x + 9 = 0$ has roots α and β.

(a) Show that $\alpha^2 + \beta^2 = \frac{13}{4}$.

For Examiner's Use

[3]

(b) The quadratic equation $px^2 - qx + r = 0$, where p, q and r are positive integers, has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. Find possible values for p, q and r.

For Examiner's Use

..... [4]

8 The parametric equations of a curve are

$$x = \frac{1}{\tan^2 t}, \; y = \cos 2t \; .$$

For Examiner's Use

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ in terms of t and hence show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\sin^4 t$.

[6]

(b) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of sin t.

For Examiner's Use

..... [3]

9 **(a)** Find the number of different arrangements of the letters

S T A T I S T I C S .

For Examiner's Use

..... [2]

(b) Find the number of different arrangements of the letters

S T A T I S T I C S

in which the Ts are together.

..... [2]

(c) An arrangement of the letters

S T A T I S T I C S

is picked at random. Given that the three Ts are together, find the probability that the two Is are not together.

For Examiner's Use

..... [4]

10 (a) Use the substitution $t = \tan\frac{\theta}{2}$ to show that

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin\theta}\,\mathrm{d}\theta = \int_0^1 \frac{2}{(1+t)^2}\,\mathrm{d}t\,.$$

For Examiner's Use

[5]

(b) Hence use integration to find the value of

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin\theta}\,\mathrm{d}\theta\,.$$

For Examiner's Use

..... [3]

11 The polynomial p(z) is defined by

$$\mathrm{p}(z) = z^3 + 5z^2 + mz + 16,$$

where m is a constant. It is given that $(z + 1)$ is a factor of p(z).

For Examiner's Use

(a) Show that $m = 20$.

[2]

(b) **Showing all your working**, find the three roots of p(z) = 0.

.....

[5]

(c) Hence, showing all your working, deduce that $z = 1 - i\sqrt{3}$ is one root of

$$p(z^2) = 0 ,$$

and find the remaining roots.

For Examiner's Use

..... [6]

12 The points O, A, B and C are the vertices of a tetrahedron. The position vectors of A, B and C relative to O are **a**, **b** and **c** respectively.

For Examiner's Use

(a) Show that the area of triangle OAB is equal to $\frac{1}{2}|\mathbf{a} \times \mathbf{b}|$.

[2]

It is given that $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{c} = 2\mathbf{j} + 6\mathbf{k}$.

(b) Find the exact value of the area of the triangle OAB.

..... [5]

(c) Find the cartesian equation of the plane containing the triangle OAB.

For Examiner's Use

..... [1]

(d) Find the exact value of the distance of the vertex C from the triangle OAB.

..... [3]

(e) Find the exact value of the volume of the tetrahedron.

[The volume of a tetrahedron $= \frac{1}{3} \times$ base area $\times$ height.]

..... [2]

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