Cambridge International Examinations
In collaboration with Nazarbayev Intellectual Schools, Kazakhstan
Grade 12

MATHEMATICS **Grade 12**

Paper 2 **May 2014**

MARK SCHEME

Maximum Mark: 90

This document consists of **8** printed pages.

NIS_G12_MATHS_02_MS/8RP

Marks awarded

- The number of marks awarded for each part of the question should be recorded in the 'For Examiner's Use' column at the right side of the page using the annotations indicated in the mark scheme e.g. M1 A1
- Half marks cannot be awarded.
- The total number of marks should be added for each question and written on the front of the question paper, added up to give the final total for the paper.
- If a question instructs the candidates to use a particular method then that method must be used.
- In other questions any valid alternative method is acceptable, and candidates should be awarded equivalent marks for reaching a comparable stage in their solution.
- Particular care should be taken when marking questions where the working leads to a given solution – the candidate must provide a full justification of the result.
- If a question requires an exact solution then the candidate must use exact values throughout their working.

Annotations and abbreviations

M Marks are awarded for using a correct method and are not lost for purely numerical errors.

A Marks are awarded for an accurate answer and depend on the preceding M marks. Therefore M0 A1 cannot be awarded.

B Marks are independent of M marks and are awarded for a correct final answer or correct intermediate stage.

DM or **DB** (or dep*) is used to indicate that a particular **M** or **B** mark is dependent on earlier **M** or **B** (asterisked) mark in the mark scheme.

Where follow through **(ft)** is indicated in the mark scheme, marks can be awarded where the candidate's work follows correctly from a previous answer, whether or not it was correct.

oee: or exact equivalent

ee: subtract 1 mark for each error (up to maximum marks available for that part)

Question	Answer	Mark	Additional Guidance
1	Use of $\log_3 3 = 1$ $\log_3 3\left(x^2+1\right) = \log_3 10x$ $3x^2 - 10x + 3 = (3x-1)(x-3) = 0$ $x = \frac{1}{3}$ or $x = 3$	B1 M1 DM1 A1 [4]	 Apply at least one log law correctly Solve a 3 term quadratic for x
2 (a)	$1 + \frac{1}{2} \times (-3x)$ ... $\ldots \frac{\frac{1}{2}\times\frac{-1}{2}}{2}(-3x)^2 + \frac{\frac{1}{2}\times\frac{-1}{2}\times\frac{-3}{2}}{3\times 2}(-3x)^3$ $1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3$	M1 A1 A1 [3]	First two terms (unsimplified) Correct unsimplified Or decimal equivalent
2 (b)	$1 - \frac{3}{2} \times 0.2 - \frac{9}{8} \times 0.04 - \frac{27}{16} \times 0.008$ **or** $1 - 3 \times 0.1 - \frac{9\times 0.01}{2} - \frac{27\times 0.001}{2}$ $= 0.6415 \approx 0.64$	M1 A1 [2]	Must have working. 0.6324 suggests M0
3 (a)	$2\cos x + 2\cos^2 x - 1 = 2$ $2\cos^2 x + 2\cos x - 3 = 0$ $\cos x = \frac{-2 \pm \sqrt{4+24}}{4}$ $= \frac{-1+\sqrt{7}}{2}$ only	M1 M1 A1 [3]	Use of double angle formula oee.
3 (b)	$x = 34.6$ and $x = 325.4$	M1 A1 [2]	One correct value for x both correct

4	$\mathbf{B}^{-1}=\frac{1}{9}\begin{pmatrix}2 & -1\\ -1 & 5\end{pmatrix}$ $\mathbf{M}=\frac{1}{9}\begin{pmatrix}a & 1\\ 3 & 1\end{pmatrix}\begin{pmatrix}2 & -1\\ -1 & 5\end{pmatrix}$ $=\frac{1}{9}\begin{pmatrix}2a-1 & -a+5\\ 5 & 2\end{pmatrix}$ $=\begin{pmatrix}\frac{2a-1}{9} & \frac{-a+5}{9}\\ \frac{5}{9} & \frac{2}{9}\end{pmatrix}$	B1 B1 M1 A2 [5]	Divide by correct determinant Transformation of matrix correct Post multiply by the inverse – one term correct -1 each error (max 2)
OR 4	$\begin{pmatrix}x & y\\ z & t\end{pmatrix}\begin{pmatrix}5 & 1\\ 1 & 2\end{pmatrix}=\begin{pmatrix}a & 1\\ 3 & 1\end{pmatrix}$ $\Rightarrow 5x+y=a,\ \ x+2y=1$ and $5z+t=3,\ z+2t=1$ $\mathbf{M}=\begin{pmatrix}\frac{2a-1}{9} & \frac{-a+5}{9}\\ \frac{5}{9} & \frac{2}{9}\end{pmatrix}$	B1 B1 M1 A2 [5]	One pair of simultaneous equations Second pair Solve for x & y in terms of a -1 each error (max 2)
5 (a)	$7\times 8.5-6\times 9$ 5.5	M1 A1 [2]	New total – old total
5 (b)	Var $=\frac{\sum x^2}{6}-81$ $\sum x^2=498$ New var $\frac{498+5.5^2}{7}-8.5^2$ $=3.21$ oee	M1 A1 M1 A1 [4]	Equation in $\sum x^2$ For their 5.5 & 498 Or better

6	$\int e^y dy = \int xe^{2x} dx$ $\int xe^{2x} dx = \frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x} dx \left(+C\right)$ $= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} \left(+C\right)$ $e^y = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$ $e = -\frac{1}{4} + C$ $e^y = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + \frac{1}{4} + e$ $y = \ln\left(\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + \frac{1}{4} + e\right)$	M1* M1 A1 A1 DM1* A1 A1 [7]	Separate correctly and attempt to integrate both sides. Integration by parts Correct parts Condone C missing Use $x = 0, y = 1$ to evaluate C. Correct equation in x & y
7 (a)	$\alpha + \beta = -\frac{7}{2}$, $\alpha\beta = \frac{9}{2}$ $\alpha^2 + \beta^2 = \left(\alpha + \beta\right)^2 - 2\alpha\beta$ $\frac{49}{4} - 9 = \frac{13}{4}$	B1 M1 A1 [3]	One correct statement *Answer given*
OR 7 (a)	$\alpha, \beta = \frac{-7 \pm \sqrt{49-72}}{4} = \frac{-7 \pm i\sqrt{23}}{4}$ $\alpha^2 + \beta^2 = \frac{49 - 23 - 14i\sqrt{23} + 49 - 23 + 14i\sqrt{23}}{16}$ $= \frac{13}{4}$	B1 M1 A1 [3]	Square and add *Answer given*
7 (b)	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{13}{4} \times \frac{2}{9} \left(= \frac{13}{18}\right)$ $\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$ $x^2 - \frac{13}{18}x + 1 = 0$ or $18x^2 - 13x + 18 = 0$ $p = 18, q = 13, r = 18$ or equivalent	B1 B1 M1 A1 [4]	Seen or implied For their sum & product

8 (a)	$\frac{dx}{dt} = -2\tan^{-3} t \times \sec^2 t$ $\frac{dy}{dt} = -2\sin 2t$ $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\tan^{-3} t \sec^2 t}$ $= \frac{-4\sin t \cos t \sin^3 t \cos^2 t}{-2\cos^3 t}$ $= 2\sin^4 t$	B1 B1 B1 M1 M1 A1 [6]	$k\tan^{-3} t \sec^2 t$ $-2\tan^{-3} t \sec^2 t$ Chain rule Double angle formula *Answer given*
8 (b)	$\frac{d^2y}{dx^2} = 8\sin^3 t \cos t \times \frac{\sin^3 t}{-2\cos t}$ $= -4\sin^6 t$	B1 M1 A1 [3]	Derivative of $\sin^4 t$ with respect to t Chain rule (with their $\frac{dx}{dt}$)
9 (a)	$\frac{10!}{3!3!2!}$ $= 50400$	M1 A1 [2]	
9 (b)	$\frac{8!}{3!2!}$ $= 3360$	M1 A1 [2]	
9 (c)	$\frac{6!}{2!} \times {}^7C_2$ $= 2520$ $\frac{2520}{3360}\left(= \frac{3}{4}\right)$ or equivalent	M1 A1 M1 A1 [4]	Is separate Correct use of their values
OR 9 (c)	$\frac{7!}{3!} = 840$ $1 - \frac{840}{3360} = \frac{2520}{3360}\left(= \frac{3}{4}\right)$ or equivalent	M1 A1 M1 A1 [4]	Is together Correct use of their values

10 (a)	$\dfrac{\mathrm{d}t}{\mathrm{d}\theta}=\dfrac{1}{2}\sec^2\left(\dfrac{\theta}{2}\right)$ $\displaystyle\int \frac{1}{1+\frac{2t}{1+t^2}}\frac{2}{1+t^2}\,\mathrm{d}t$ $\theta=\frac{\pi}{2}, t=1\ \ \theta=0, t=0$ $\displaystyle\int_0^1 \frac{2}{(1+t)^2}\,\mathrm{d}t$	B1 M1 A1 B1 A1 [5]	Or equivalent Substitute t for θ Correct unsimplified Deal with the limits *Answer given*
10 (b)	$\left[\dfrac{-2}{1+t}\right]_0^1$ $=\dfrac{-2}{2}+2=1$	M1 DM1 A1 [3]	To include $(1+t)^{-1}$ Use correct limits correctly
11 (a)	$\mathrm{p}(-1)=0=-1+5-m+16$ $m=20$	M1 A1 [2]	Or use division of polynomials
11 (b)	$z=-1$ $\mathrm{p}(z)=(z+1)\left(z^2+4z+16\right)$ $z=\dfrac{-4\pm\sqrt{16-64}}{2}=-2\pm\mathrm{i}\sqrt{12}=-2\pm2\sqrt{3}\mathrm{i}$	B1 M1 A1 M1 A1 [5]	 Factorization by inspection or division Solve 3 term quadratic for z.
11 (c)	$\pm\mathrm{i}$ $\left(1-\sqrt{3}\mathrm{i}\right)^2=1-2\sqrt{3}\mathrm{i}-3$ $=-2-2\sqrt{3}\mathrm{i}$ = square root of result from **(b)** and hence root of $\mathrm{p}\left(z^2\right)=0$ $z=1+\sqrt{3}\mathrm{i}$ $z=-1+\sqrt{3}\mathrm{i}$ $z=-1-\sqrt{3}\mathrm{i}$	B1 M1 A1 B1 B1 B1 [6]	 Square the given value OR an exact method for $\sqrt{-2-2\sqrt{3}\mathrm{i}}$ Correct deduction of **given result**. Complex conjugate will be a root Negatives will be roots

12 (a)	Area $=\frac{1}{2}\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$ $\mathbf{a}\times\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\cdot\mathbf{n}$ **or** $\lvert\mathbf{a}\times\mathbf{b}\rvert = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$	B1 B1 [2]	Reference to $\frac{1}{2}ab\sin C$ Quote formula for vector product and deduce **given result**.
12 (b)	$\mathbf{a}\times\mathbf{b}=\mathbf{i}(-4-6)-\mathbf{j}(8-3)+\mathbf{k}(4+1)$ Area $=\frac{1}{2}\sqrt{10^2+5^2+5^2}$ $=\frac{1}{2}\times 5\sqrt{6}$	M1 A1 A1 M1 A1 [5]	Clear attempt at vector product with one term correct. 2 terms correct All correct Modulus Or exact equivalent
OR **12 (b)**	$\cos(\angle AOB)=\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}=\frac{2-2+12}{\sqrt{14}\times\sqrt{21}}$ $=\frac{12}{\sqrt{14}\sqrt{21}}$ Area $=\frac{1}{2}\sqrt{14}\sqrt{21}\times\frac{\sqrt{14\times 21-144}}{\sqrt{14}\sqrt{21}}$ $=\frac{5\sqrt{6}}{2}$	M1 A1 A1 M1 A1 [5]	Use scalar product to find an angle Correct scalar product Or exact equivalent Area of triangle Or exact equivalent
12 (c)	Plane OAB: $2x+y-z=0$	B1 [1]	their **n**
12 (d)	$\mathbf{c}\cdot\mathbf{n} = 2-6 = -4$ Distance $= \lvert\mathbf{c}\cdot\hat{\mathbf{n}}\rvert = \frac{4}{\sqrt{6}} = 2\frac{\sqrt{6}}{3}$	M1 M1 A1 [3]	Scalar product with their **n** Correct method for distance Or exact equivalent [Method marks may be through substitution in formula included in Formulae Booklet.]
12 (e)	Volume $=\frac{1}{3}\times$ base area $\times$ height $=\frac{1}{3}\times\frac{1}{2}5\sqrt{6}\times\frac{2\sqrt{6}}{3}=\frac{10}{3}$	M1 A1 [2]	Correct method with their values Correct answer only