CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS   **Grade 12**

Paper 3   **May 2014**

                                                                       2 hours

Candidates answer on the Question Paper.

Additional Materials: Calculator
                       Geometrical Instruments
                       List of Formulae and Statistical Tables   **12MATH/03**

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do **not** use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.

Electronic calculators may be used.

You may lose marks if you do not show your working or if you do not use appropriate units.

Give non-exact numerical answers correct to 3 significant figures and angles correct to 0.1° unless a different degree of accuracy is specified in the question.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
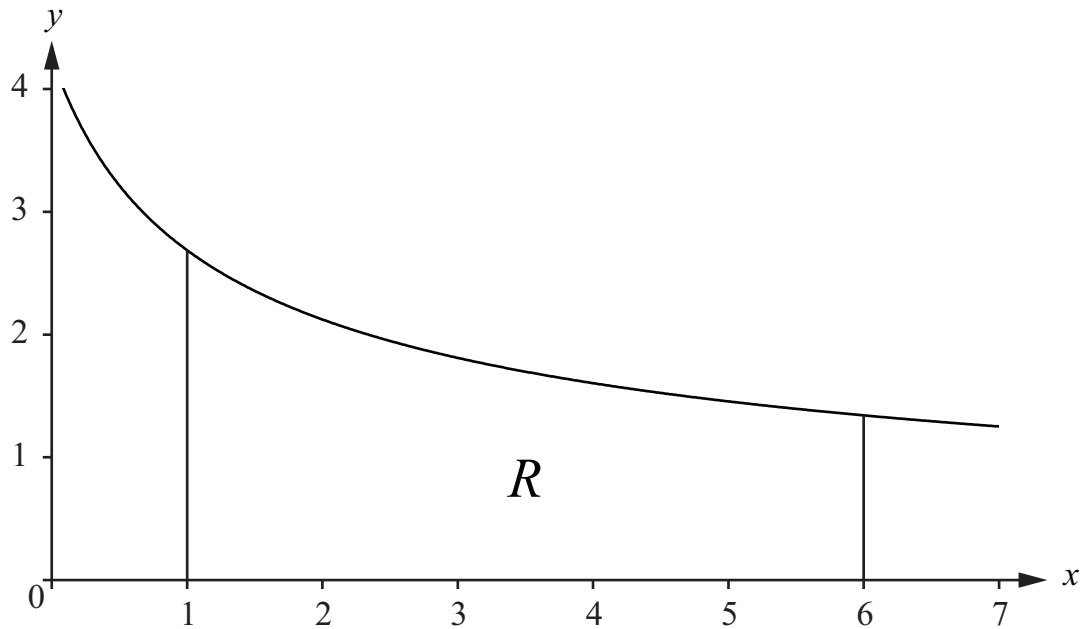
The total number of marks for this paper is 80.

For Examiner's Use	
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This document consists of **14** printed pages and **2** blank pages.

1 The diagram shows the region R enclosed by the curve $y = \dfrac{6}{\sqrt{3x+2}}$ and the lines

$x = 1$, $x = 6$ and $y = 0$.

A carpenter forms a solid by rotating R through $360°$ about the x-axis.
Show that the volume of the solid is $24\pi \ln 2$.

[5]

2 A firm sells x computers per week.
The income from selling x computers is given by $I(x) = 1200x - 2x^2$.
The cost of making x computers is given by $C(x) = x^3 - 72.5x^2 + 1470x + 2000$.
$I(x)$ and $C(x)$ are in dollars.

(a) It is given that the profit is the difference between the income and cost. You should assume that the profit function is continuous. Use differentiation to find the x values for which this function is stationary.

.... [6]

(b) Using the second derivative or otherwise, show that profit is maximised if the firm sells 45 computers a week and state this profit.

.... [4]

3 Coloured lights on a computer screen flash in sequence. The lights are in positions corresponding to the roots of the equation $z^3 + 8 = 0$, as represented on an Argand diagram.

(a) Solve the equation $z^3 + 8 = 0$, giving the roots in the form $x + iy$.

.... [5]

(b) Draw an Argand diagram showing these roots.

[1]

4   The time spent per day on homework by students in a college is modelled by a normal distribution with mean 200 minutes and standard deviation 20 minutes.

   (a) Find the probability that a randomly selected student will spend more than 4 hours on homework.

   .... [4]

   (b) Find the probability that a randomly selected student will spend between 180 and 200 minutes on homework.

   .... [4]

5 **(a)** Use De Moivre's theorem to show that $\cos 5x \equiv 16\cos^5 x - 20\cos^3 x + 5\cos x$.

[5]

(b) A curve with equation $y = 16\cos^5 x - 20\cos^3 x + 5\cos x + 1$ for $0 \leq x \leq 2\pi$ forms a design on a piece of pottery. Determine the values of x for which $y = 0$.

[3]

6 It is known that 25% of all senior pupils in a school are in favour of a proposal to increase the time spent on homework. Five senior pupils are selected at random. Find the probability that the proposal is favoured by

(a) at least one of the five senior pupils,

.... [3]

(b) a majority of the five senior pupils.

.... [3]

7 A sculpture is based on three planes intersecting at a single point. The planes have equations

$$2x + y + 5z = 30, \qquad x + 2y - z = 2, \qquad -3x + y + z = 5.$$

It is given that $\mathbf{A} = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 2 & -1 \\ -3 & 1 & 1 \end{pmatrix}$.

(a) Show that the determinant of **A** is 43.

[2]

(b) Use Cramer's method to solve the following simultaneous equations.

$$2x + y + 5z = 30$$

$$x + 2y - z = 2$$

$$-3x + y + z = 5$$

$x = \underline{\quad\quad\quad}$, $y = \underline{\quad\quad\quad}$, $z = \underline{\quad\quad\quad}$ [5]

(c) Write down the coordinates of the single point belonging to all three planes of the sculpture.

.... [1]

8 A sporting event consists of shooting arrows at a plane surface. Three points lie in the plane surface. Their position vectors relative to an origin O are

$$\mathbf{r}_1 = 3\mathbf{i} + \mathbf{j} + 5\mathbf{k}, \qquad \mathbf{r}_2 = 12\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \qquad \mathbf{r}_3 = 9\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}.$$

(a) Show that $(\mathbf{r}_2 - \mathbf{r}_1) \times (\mathbf{r}_3 - \mathbf{r}_1) = 6\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$.

[4]

(b) By considering the dot (scalar) product of $(\mathbf{r} - \mathbf{r}_1)$ and $6\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$ or otherwise, find the cartesian equation of the plane surface which contains the three points.

.... [4]

For Examiner's Use

An arrow is fired at the plane surface and intersects it at *B*. The line of the arrow passes through the point with position vector $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ and has direction vector $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(c) Write down the vector equation of the line of the arrow in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$.

.... [1]

(d) Find the coordinates of *B*.

.... [3]

9 Newton's law of cooling states that the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air.

On a particular day, the temperature of the air is 30 degrees Celsius.

It is given that at time t minutes, the temperature of the substance is T degrees Celsius.

(a) Explain why $\dfrac{dT}{dt} = -k\,(T - 30)$ where k is a positive constant.

[3]

(b) Hence show that $T = A e^{-kt} + 30$ where A and k are positive constants.

[4]

(c) Given that the substance cools from 80 degrees Celsius to 60 degrees Celsius in 10 minutes, calculate the time required to cool from 80 degrees Celsius to 35 degrees Celsius.

.... [4]

10 (a) Eight candidates are available to fill four positions on a committee. Five candidates are men and three are women. If each combination of candidates is equally likely to be chosen, find the probability that no woman will be chosen.

.... [3]

(b) A factory employs a large number of people, $\dfrac{1}{12}$ of whom are women.

Employees are selected at random to work on a special project. Find the probability that the first woman selected is the seventh person chosen.

.... [3]

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