Cambridge International Examinations
In collaboration with Nazarbayev Intellectual Schools, Kazakhstan
Grade 12

MATHEMATICS **Grade 12**

Paper 3 **May 2014**

MARK SCHEME

Maximum Mark: 80

This document consists of **8** printed pages.

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[Turn over

Marks awarded

- The number of marks awarded for each part of the question should be recorded in the 'For Examiner's Use' column at the right side of the page using the annotations indicated in the mark scheme e.g. M1 A1
- Half marks cannot be awarded.
- The total number of marks should be added for each question and written on the front of the question paper, added up to give the final total for the paper.
- If a question instructs the candidates to use a particular method then that method must be used.
- In other questions any valid alternative method is acceptable, and candidates should be awarded equivalent marks for reaching a comparable stage in their solution.
- Particular care should be taken when marking questions where the working leads to a given solution – the candidate must provide a full justification of the result.
- If a question requires an exact solution then the candidate must use exact values throughout their working.

Annotations and abbreviations

M Marks are awarded for using a correct method and are not lost for purely numerical errors.

A Marks are awarded for an accurate answer and depend on the preceding M marks. Therefore M0 A1 cannot be awarded.

B Marks are independent of M marks and are awarded for a correct final answer or correct intermediate stage.

DM or **DB** (or dep*) is used to indicate that a particular **M** or **B** mark is dependent on earlier **M** or **B** (asterisked) mark in the mark scheme.

Where follow through **(ft)** is indicated in the mark scheme, marks can be awarded where the candidate's work follows correctly from a previous answer, whether or not it was correct.

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Question	Answer	Mark	Additional Guidance
1	Attempt use of $(\pi)\int_1^6 \frac{36}{3x+2}dx$	M1	
	Obtain $12(\pi)\ln(3x+2)$	A1	
	Use limits on their log integral in the correct order	M1	
	Attempt use of at least one log law	M1	
	Obtain $24\pi\ln 2$	A1 [5]	Answer is given so check all working
2 (a)	Attempt $P(x) = I(x) - C(x)$	M1	
	Obtain $-x^3 + 70.5x^2 - 270x - 2000$	A1	
	Attempt to differentiate their $P(x)$ function	M1	
	Obtain $-3x^2 + 141x - 270$	A1	
	Put $\frac{dy}{dx} = 0$ and attempt to solve their quadratic	M1	
	Obtain $x = 45$ and 2	A1 [6]	
2 (b)	Attempt to obtain their second derivative	M1	Allow equivalent methods, e.g. testing the sign of the first derivative either side of 45 or finding P(x) values either side of 45.
	Attempt to substitute their x value	M1	
	Obtain -129 and conclude < 0 so max	A1	
	State 37487.5 (dollars)	A1 [4]	

3 (a)	State or imply $z = 8(\cos(\pi + 2k\pi) + i\sin(\pi + 2k\pi))$ Attempt to cube root 8 and divide angle expressions by 3 Obtain two of $2(\cos\pi + i\sin\pi)$ $2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ $2(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$ Attempt to turn trig form to algebraic form Obtain $1 + \sqrt{3}i$, $1 - \sqrt{3}i$ and -2	B1 M1 A1 DM1 A1 [5]	Alternative methods should be allowed
OR 3(a)	State $z + 2$ is a factor Attempt division of $z^3 + 8$ by $z + 2$ Obtain $z^2 - 2z + 4$ Use the quadratic formula to solve their quadratic Obtain $1 + \sqrt{3}i$, $1 - \sqrt{3}i$ and -2	B1 M1 A1 DM1 A1 [5]	
3 (b)	Clear drawing showing points $(-2, 0)$, $(1, \sqrt{3})$, $(1, -\sqrt{3})$	B1 [1]	
4 (a)	Use $z = \frac{x - 200}{20}$ Obtain $z = 2$ Attempt $P(x > 240) = 1 - P(z \leq 2)$ Obtain 0.0228	M1 A1 M1 A1 [4]	
4 (b)	Use $z = \frac{x - 200}{20}$ Obtain $z = 0$ and $z = -1$ Attempt $P(-1 < z < 0) = P(z \leq 1) - 0.5$ Obtain 0.341(3)	M1 A1 M1 A1 [4]	

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5 (a)	State $(\cos x + \text{i}\sin x)^5 = \cos 5x + \text{i}\sin 5x$ Attempt to expand $(\cos x + \text{i}\sin x)^5$ Obtain at least $\cos^5 x - 10\cos^3 x \sin^2 x + 5\cos x \sin^4 x$ Replace $\sin^2 x$ with $(1 - \cos^2 x)$ in their expansion Obtain $16\cos^5 x - 20\cos^3 x + 5\cos x$	B1 M1 A1 M1 A1 [5]	Obtain at least two unsimplified terms Must be seen correct at least once Answer given in question so full details must be given.
5 (b)	State or imply $16\cos^5 x - 20\cos^3 x + 5\cos x + 1 = \cos 5x + 1$ Attempt correct method to solve $\cos 5x = -1$ Obtain $\frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$	B1 M1 A1 [3]	Use correct order of operations
6 (a)	State or imply Bin (5, 0.25) Use $P(X \geq 1) = 1 - P(0)$ Obtain 0.763 (0.7626953…)	B1 M1 A1 [3]	Accept P(X = 1) +.....+ P(X = 5)
6 (b)	State $P(X \geq 3)$ Attempt ${}^5C_3\ (0.25)^3(0.75)^2 + {}^5C_4\ (0.25)^4(0.75) + (0.25)^5$ Obtain 0.104 (0.1035156…)	B1 M1 A1 [3]	Binomial Coefficient or equivalent fraction Accept equivalent fractions
7 (a)	Attempt expansion $2\begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix} - 1\begin{vmatrix}1 & -1\\ -3 & 1\end{vmatrix} + 5\begin{vmatrix}1 & 2\\ -3 & 1\end{vmatrix} =$ $2(2 + 1) - (1 - 3) + 5\ (1 + 6)$ or better $(6 + 2 + 35)$	M1 A1 [2]	Obtain at least e.g $2\lvert...\rvert - 1\lvert...\rvert + 5\lvert...\rvert$ if expanding by the first row and similarly for other expansions Answer given in the question

7 (b)	Use correct method for obtaining $\lvert A_1\rvert$ $\lvert A_2\rvert$ and $\lvert A_3\rvert$ Attempt to divide their $\lvert A_1\rvert$ $\lvert A_2\rvert$ and $\lvert A_3\rvert$ by 43 Obtain $x = 1$, $y = 3$ and $z = 5$	M1 M1 A3 [5]	$\lvert A_1\rvert = \begin{vmatrix} 30 & 1 & 5 \\ 2 & 2 & -1 \\ 5 & 1 & 1 \end{vmatrix}$ $\lvert A_2\rvert = \begin{vmatrix} 2 & 30 & 5 \\ 1 & 2 & -1 \\ -3 & 5 & 1 \end{vmatrix}$ $\lvert A_3\rvert = \begin{vmatrix} 2 & 1 & 30 \\ 1 & 2 & 2 \\ -3 & 1 & 5 \end{vmatrix}$
7 (c)	State (1, 3, 5)	A1ft [1]	
8 (a)	Obtain $\mathbf{r}_2 - \mathbf{r}_1 = 9\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ and $\mathbf{r}_3 - \mathbf{r}_1 = 6\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ State $\begin{vmatrix} i & j & k \\ 9 & 1 & -4 \\ 6 & 2 & -2 \end{vmatrix}$ Attempt to evaluate Obtain $6\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$	B1 B1 M1 A1 [4]	Note that this answer is given in the question

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8 (b)	State $\begin{pmatrix} x-3 \\ y-1 \\ z-5 \end{pmatrix}$ Attempt dot product with $6\mathbf{i} - 6\mathbf{j} + 12\mathbf{k} = 0$ Attempt simplification of unsimplified answer Obtain $x - y + 2z = 12$	B1 M1 M1 A1 [4]	 Must show sum of products Must expand brackets and collect terms
8 (c)	State $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$	B1 [1]	
8 (d)	Attempt to solve $\left[\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}\right] \bullet \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 12$ Obtain $\lambda = -1.5$ Obtain $(-2.5, 1.5, 8)$	M1 A1 A1 [3]	Alternative methods should be allowed Accept any equivalent form
9 (a)	State rate implies $\frac{dT}{dt}$ Difference between 30 and substance given by $(T - 30)$ Cooling proportional to a constant factor given by $-k$	B1 B1 B1 [3]	
9 (b)	State $\int \frac{1}{T-30} dT = -\int k dt$ Attempt integration of both sides Obtain $T - 30 = e^{-kt + c}$ State or imply $e^c = A$ and conclude $T = Ae^{-kt} + 30$	B1 M1 A1 A1 [4]	 Note that this answer is given in the question.

9 (c)	State $t = 0$ so $80 = A + 30$ and $A = 50$ Attempt to solve $60 =$ their $Ae^{-10k} + 30$ Obtain k = 0.05108 Obtain $t = 45.1$ minutes	B1 M1 A1 A1 [4]	Alternative methods should be allowed or exact equivalent
10 (a)	State ${}^5C_4$ State ${}^8C_4$ Divide and obtain $\frac{1}{14}$ or 0.0714	B1 B1 B1 [3]	Alternative methods should be allowed
10 (b)	State or imply Bernoulli trials (geometric distribution) Use $P(X = 7) = (\frac{1}{12})(\frac{11}{12})^6$ Obtain 0.0494 or exact equivalent.	B1 M1 A1 [3]	