AEO “Nazarbayev Intellectual Schools”
Cambridge International Examinations

MATHEMATICS **Grade 12**

Paper 2 **May 2016**

MARK SCHEME

Maximum Mark: 90

This document consists of **10** printed pages.

CAMBRIDGE
International Examinations

Marks awarded

- The number of marks awarded for each part of the question should be recorded in the 'For Examiner's Use' column at the right side of the page using the annotations indicated in the mark scheme e.g. M1 A1
- Half marks cannot be awarded.
- The total number of marks should be added for each page and written on the front of the question paper, added up to give the final total for the paper.
- If a question instructs the candidates to use a particular method then that method must be used.
- In other questions any valid alternative method is acceptable, and candidates should be awarded equivalent marks for reaching a comparable stage in their solution.
- Particular care should be taken when marking questions where the working leads to a given solution – the candidate must provide a full justification of the result.
- If a question requires an exact solution then the candidate must use exact values throughout their working.

Annotations and abbreviations

M Marks are awarded for using a correct method and are not lost for purely numerical errors.

A Marks are awarded for an accurate answer and depend on the preceding M marks. Therefore M0 A1 cannot be awarded.

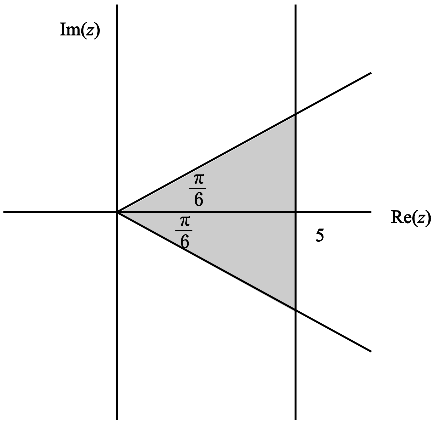
B Marks are independent of M marks and are awarded for a correct final answer or correct intermediate stage.

DM or **DB** (or dep*) is used to indicate that a particular **M** or **B** mark is dependent on earlier **M** or **B** (asterisked) mark in the mark scheme.

Where follow through **(ft)** is indicated in the mark scheme, marks can be awarded where the candidate's work follows correctly from a previous answer, whether or not it was correct.

oee: or exact equivalent

ee: subtract 1 mark for each error (up to maximum marks available for that part)

Question	Answer	Mark	Additional Guidance
1	Differentiate the algebraic sum of functions $\frac{\mathrm{d}y}{\mathrm{d}x}=2x-\frac{1}{x^2}$ $x=2,\ \frac{\mathrm{d}y}{\mathrm{d}x}=4-\frac{1}{4}=3\frac{3}{4}$	M1 A1 A1 [3]	Evidence of correct method of differentiation Accept equivalent forms. Accept exact equivalents
2(a)		B1 B1 B1 [3]	Correct vertical line Correct lines through origin with angles looking or marked as correct. If the angles are labeled and they are not more than 45^0 award B1. If the angles are not labeled and they are not more than 45^0 award B1. In other cases B1 is not awarded. Correct region shaded
2(b)	Max $\operatorname{mod}(z)=\sqrt{5^2+\left(5\tan\frac{\pi}{6}\right)^2}$ $=\sqrt{25+\frac{25}{3}}=\sqrt{\frac{100}{3}}\ \left(=\frac{10}{3}\sqrt{3}\right)$	M1 A1 [2]	 Accept exact equivalents and 5.77 or better
3(a)	$\alpha+\beta=-2,\ \alpha\beta=8$ Use of $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=4-16=-12$	B1 M1 A1 [3]	 With their values for $\alpha+\beta$ and $\alpha\beta$ **Answer given** – must see supporting working
3(a) alt	Roots of the quadratic are $-1 \pm \mathrm{i}\sqrt{7}$ $\alpha^2+\beta^2=\left(-1-\mathrm{i}\sqrt{7}\right)^2+\left(-1+\mathrm{i}\sqrt{7}\right)^2$ $=\left(-6+2\sqrt{7}\mathrm{i}\right)+\left(-6-2\sqrt{7}\mathrm{i}\right)=-12$	B1 M1 A1 [3]	 Square and add. **Answer given** – must see supporting working
3(b)	$(2\alpha)^2+(2\beta)^2=4\left(\alpha^2+\beta^2\right)=-48,\ p=48$ $(2\alpha)^2\times(2\beta)^2=16\alpha^2\beta^2=16\times64=1024$	B1 B1ft	 For $16\times(\text{their } \alpha\beta)^2$

		[2]	Accept answers implied by $x^2 + 48x + 1024 = 0$

4	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\mathrm{e}^{-x}(1+x^2)-2x\mathrm{e}^{-x}}{(1+x^2)^2}$	M1 A1	Use correct product or quotient rule Any exact equivalent
	At stationary point $\frac{-\mathrm{e}^{-x}(1+x^2)-2x\mathrm{e}^{-x}}{(1+x^2)^2} = 0$	M1	Equate to zero the product. This can be missed.
	$x^2+2x+1=(x+1)^2=0$ $x=-1$	A1	
	$y=\frac{\mathrm{e}}{2}$	A1	Any exact equivalent
		[5]	
5	$\frac{1-t^2}{1+t^2}+2\frac{2t}{1+t^2}=1$	M1A1	Must be using the t-substitution $1-tg^2\frac{\theta}{2}+4tg\frac{\theta}{2}=1+tg^2\frac{\theta}{2}$ is accepted.
	$\Rightarrow 1-t^2+4t=1+t^2,\quad 2t^2-4t=0$	M1	Other methods are not accepted.
	$t(t-2)=0,$		
	$\frac{1}{2}\theta=0$ or $\frac{1}{2}\theta=\tan^{-1}2$	M1	
	$\theta=0,\ \theta=126.9°(127°)$	A1	Answer is accepted only in degrees.
		[5]	

6(a)	$2+2\sqrt{3}\mathrm{i}=4\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}\right)$ $r=4,\quad \theta=\frac{\pi}{3}$	B1 B1 [2]	Correct θ Correct notation of complex number in trigonometric form.
6(b)	Modulus $4\times3=12$ Argument $\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$	B1ft B1ft [2]	$3\times$ their r Their $\theta-\frac{\pi}{4}$
6(c)	$n\times\frac{7}{12}\pi$ is a multiple of π $n=12$	M1 A1 [2]	
7(a)	$\mathrm{p}(-1)=-2+a-b-6=0\quad(a-b=8)$ $\mathrm{p}(2)=16+4a+2b-6=12\quad(4a+2b=2)$ $\Rightarrow 6a=18,\ a=3$ $b=-5$	M1 A1 M1 A1 [4]	Use remainder/factor theorem to form one correct (unsimplified) equation. 2nd (unsimplified) equation Solve for a or b Both correct Allow equivalent methods
7(b)	Divide by $(x+1)$ and reach at least $2x^2+kx...$ Quadratic factor $(2x^2+x-6)$ $\mathrm{p}(x)=(x+1)(2x-3)(x+2)$	M1 A1 A1 [3]	Or multiply $(x+1)$ by (Ax^2+Bx+C) and compare coefficients, or equivalent Allow equivalent methods
8(a)	$\frac{2\tan x}{1+\tan^2 x}=\frac{2\frac{\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}}$ $=\frac{2\frac{\sin x}{\cos x}\cos^2 x}{\cos^2 x+\sin^2 x}=2\sin x\cos x$ $=\sin 2x$	M1 M1 A1 [3]	Use $\tan x=\frac{\sin x}{\cos x}$ Use $\cos^2 x+\sin^2 x=1$ Obtain **given result** correctly

8(a) alt	$\frac{2\tan x}{1+\tan^2 x}=\frac{2\tan x}{\sec^2 x}$ $=2\frac{\sin x}{\cos x}\cos^2 x=2\sin x\cos x$ $=\sin 2x$	M1 M1 A1 [3]	Use of $1+\tan^2 x=\sec^2 x$ Use of $\tan x=\frac{\sin x}{\cos x}$ Obtain **given result** correctly
8(b)	$2\sin x\cos x=\frac{1}{3}\sin x$ $\sin x\left(\cos x-\frac{1}{6}\right)=0$, $x=0,\ \pi,\ 2\pi$ $x=1.40,\quad 4.88$ Allow M1B1M1A0 for $0°,\ 180°,\ 360°,\ 80°,\ 280°$	M1 B1 M1 A1 [4]	Use the result from (a) Accept 3.14, 6.28 Solve to find one solution of $\cos x=\frac{1}{6}$ Two correct solutions from $\cos x=\frac{1}{6}$ and no other solutions in $0\le x\le 2\pi$ $\frac{67\pi}{150}$ and $\frac{233\pi}{150}$ accepted
9(a)	$(1+ax)^6=1+6ax$ $+15a^2x^2+....$	B1 B1 [2]	B1 is awarded for the first two terms
9(b)	$\left(1+6ax+15a^2x^2+....\right)\left(1+2x+3x^2\right)$ $1+6ax+15a^2x^2+2x+12ax^2+3x^2+...$ $6a+2=b$ $15a^2+12a+3=39$ $15a^2+12a-36=0,\quad 5a^2+4a-12=0$ $(5a-6)(a+2)=0$ $a=-2,\quad b=-10$	M1 DM1 A1 M1 A1 [5]	Multiply to find all terms up to x^2 Compare coefficients Correct equations Solve a 3 term quadratic for a

10(a)	$\frac{7!}{2!2!}$ $=1260$	M1 A1 [2]	
10(b)	End in 4: $\frac{6!}{2!2!}$ End in 2: $\frac{6!}{2!}$ Total = 540	M1 A1 [2]	M1 is awarded for any of shown expressions
10(b) alt	$\frac{3}{7}$ digits are even: $\frac{3}{7}\times 1260$ $=540$	M1 A1 [2]	
10(c)	Multiples of 4 end in $12{:}\ \frac{5!}{2!}$ $32{:}\ 5!$ $52{:}\ \frac{5!}{2!}$ $24{:}\ \frac{5!}{2!}$ Total 300 $\frac{\text{Multiples of 4}}{\text{Multiples of 2}}$ $=\frac{300}{540}$ P(both) = $\left(\frac{5}{9}\right)^2$ $=\frac{25}{81}$	M1 A1 M1 A1 [4]	M1 is awarded if any three of given expressions are written. $=\frac{15}{27}=\frac{5}{9}$ Their answer squared Or equivalent (0.309 or better) Answers in percent are not accepted.

11	Volume under curve = $\int \pi y^2 \mathrm{d}x = \pi \int (x+\cos x)^2 \mathrm{d}x$ $= \pi \int (x^2 + 2x\cos x + \cos^2 x)\mathrm{d}x$ $\int x^2 \mathrm{d}x = \frac{1}{3}x^3$ $\int x\cos x \mathrm{d}x = x\sin x - \int \sin x \mathrm{d}x$ $= x\sin x + \cos x$ $\int \cos^2 x \mathrm{d}x = \int \frac{\cos 2x + 1}{2}\mathrm{d}x$ $= \frac{1}{4}\sin 2x + \frac{1}{2}x$ Volume $= \pi\left[\frac{1}{3}x^3 + 2x\sin x + 2\cos x + \frac{1}{2}x + \frac{1}{4}\sin 2x\right]$ $= \pi\left[\frac{\pi^3}{3} - 2 + \frac{\pi}{2} - 2\right]$ $= \pi\left(\frac{\pi^3}{3} + \frac{\pi}{2} - 4\right)$	M1 B1 M1 A1 A1 M1A1 A1 M1 A1 [10]	Correct method for volume but condone missing/incorrect limits at this stage Correct method for integration by parts. Correct at first stage Completed correctly Condone 2 missing for M1A1A1 Use double angle substitution Allow M1A0 with a sign error. Correct use of limits 0 and π Accept exact equivalents If π is missing use of $\int y^2 \mathrm{d}x$ can score 8/10 marks, i.e. M0B1M1A1A1M1A1A1M1A0
12(a)	$\mathbf{r} = (2+2\lambda)\mathbf{i} + \mathbf{j} + (3+\lambda)\mathbf{k}$ $2x + y - z = 2(2+2\lambda) + 1 - (3+\lambda)$ $= 2 + 3\lambda = 8, \quad \lambda = 2$ $\mathbf{r} = 6\mathbf{i} + \mathbf{j} + 5\mathbf{k}$	M1 A1 A1 [3]	M1 is awarded for two inequalities. Accept coordinates

12(b)	$\begin{pmatrix}2\\1\\3\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}2+2\mu\\1+\mu\\3-\mu\end{pmatrix}$ Meets the plane when $2(2+2\mu)+(1+\mu)-(3-\mu)=8$ $2+6\mu=8,\ \ \mu=1$ Position vector of reflection is $6\mathbf{i}+3\mathbf{j}+\mathbf{k}$ $\Rightarrow$ coordinates $(6,\ 3,\ 1)$	M1 M1 DM1 A1 [4]	Vector through the point and perpendicular to the plane Own methods which gives $\begin{pmatrix}2+2\mu\\1+\mu\\3-\mu\end{pmatrix}$ is accepted. Equation for the value of μ where the line meets the plane Use $2\times$ their μ (dependent on both preceding M marks). Accept the position vector
12(c)	$\begin{pmatrix}2\\0\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\-1\end{pmatrix}=3$ $3=\sqrt{4+0+1}\times\sqrt{4+1+1}\times\cos(90-\alpha)$ $3=\sqrt{4+0+1}\times\sqrt{4+1+1}\times\sin\alpha$ $\alpha=\sin^{-1}\left(\dfrac{3}{\sqrt{30}}\right)$ $\alpha=33.2$	M1 M1 A1 A1 [4]	Scalar product of two relevant vectors Correct use of scalar product to find α or $90-\alpha$ Correct value for trig function Correct angle. Accept 0.58 radians.
12(c) alt	Distance (2, 1, 3) to plane = $\sqrt{2^2+1^2+1^2}$ Distance (2, 1, 3) to (6, 1, 5) $=\sqrt{4^2+2^2}=\sqrt{20}$ $\sin\alpha=\dfrac{\sqrt{6}}{\sqrt{20}}(=0.547..)$ $\alpha=33.2$	M1 M1 A1 A1 [4]	Find two sides of a relevant right angled triangle Use correct trig ratio for their sides Correct (unsimplified) value Accept 0.58 radians.
13(a)	$\dfrac{dy}{dx}=\dfrac{3}{2\sqrt{x}}$ $x=9,\ \ \dfrac{dy}{dx}=\dfrac{3}{6}=\dfrac{1}{2}$	M1 A1 M1 M1	Differentiate to obtain gradient function Correct derivative Gradient at (9,9) Equation of tangent

	Equation of tangent: $\dfrac{y-9}{x-9}=\dfrac{1}{2}$ $(2y=x+9)\quad x=0\Rightarrow y=\dfrac{9}{2}$	A1 [5]	Obtain **given answer** correctly

13(b)	Area between curve & y-axis $\int_0^9 \frac{y^2}{9}\,dy = \left[\frac{y^3}{27}\right]_0^9$ $= 27$ Area of triangle $= \frac{1}{2}\times\left(9-\frac{9}{2}\right)\times 9$ Required area $= 27-\frac{81}{4}$ $= \frac{27}{4}$ (6.75)	B1 M1 A1 B1 M1 A1 [6]	$x = \frac{y^2}{9}$ Area between curve and y-axis or exact equivalent
13(b)	$S = \int_0^9 \left(\frac{1}{2}x + \frac{9}{2} - 3\sqrt{x}\right)dx$ The limits of integration is correctly written $= \left(\frac{1}{4}x^2 + \frac{9}{2}x - 2x\sqrt{x}\right)\Big\vert_0^9$ The limits are substituted correctly Obtain $\frac{27}{4}$ or 6.75	M1 B1 A1 B1 M1 A1 [6]	The difference of functions should be exactly in this order (equation of its tangent)
13(b) alt	Area of rectangle $= 9\times 9 = 81$ Area of triangle $= \frac{1}{2}\times\left(9-\frac{9}{2}\right)\times 9$ Area between curve and x-axis $\int_0^9 3\sqrt{x}\,dx = \left[2x^{\frac{3}{2}}\right]_0^9$ $= 2\times 27 = 54$ Required area $= 81-54-\frac{81}{4}$ $= 6.75$	B1 B1 M1 A1 M1 A1 [6]	Working from the x-axis For area of the trapezium $\frac{1}{2}\left(9+\frac{9}{2}\right)\times 9$ scores B2 or exact equivalent