AEO "Nazarbayev Intellectual Schools"
Cambridge International Examinations

MATHEMATICS	**Grade 12**
Paper 3	**May 2016**
MARK SCHEME	
Maximum Mark: 80	

This document consists of **9** printed pages and **1** blank page.

NIS_G12_MATHS_03_MS/4RP

CAMBRIDGE
International Examinations

[Turn over

Marks awarded

- The number of marks awarded for each part of the question should be recorded in the 'For Examiner's Use' column at the right side of the page using the annotations indicated in the mark scheme e.g. M1 A1
- Half marks cannot be awarded.
- The total number of marks should be added for each page and written on the front of the question paper, added up to give the final total for the paper.
- If a question instructs the candidates to use a particular method then that method must be used.
- In other questions any valid alternative method is acceptable, and candidates should be awarded equivalent marks for reaching a comparable stage in their solution.
- Particular care should be taken when marking questions where the working leads to a given solution – the candidate must provide a full justification of the result.
- If a question requires an exact solution then the candidate must use exact values throughout their working.

Annotations and abbreviations

M Marks are awarded for using a correct method and are not lost for purely numerical errors.

A Marks are awarded for an accurate answer and depend on the preceding M marks. Therefore M0 A1 cannot be awarded.

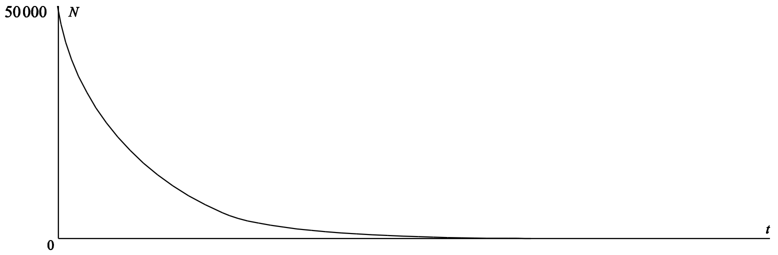
B Marks are independent of M marks and are awarded for a correct final answer or correct intermediate stage.

DM or **DB** (or dep*) is used to indicate that a particular **M** or **B** mark is dependent on earlier **M** or **B** (asterisked) mark in the mark scheme.

Where follow through **(ft)** is indicated in the mark scheme, marks can be awarded where the candidate's work follows correctly from a previous answer, whether or not it was correct.

oee: or exact equivalent

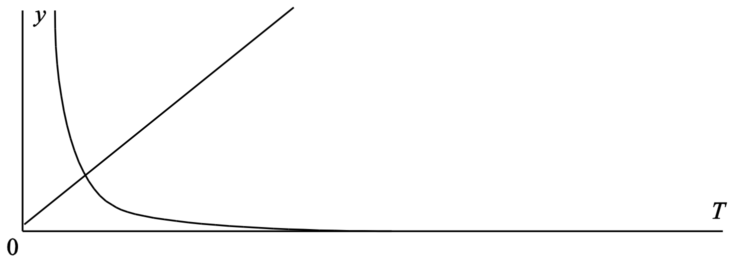
ee: subtract 1 mark for each error (up to maximum marks available for that part)

Question	Answer	Mark	Additional Guidance
1 (a)	Discrete bars or lines seen At least 5 frequencies correctly plotted All frequencies correctly plotted	M1 A1 A1 **[3]**	
1 (b)	Use [(0 ×5) + (1 × 10) + (2 × 20) + (3 × 60) + (4 × 100) + (5 × 140) + (6 × 160) + (7 × 175) + (8 × 140) + (9 × 100) + (10 × 50)] / 960 Obtain $\frac{6035}{960}$ = 6.29 (6.286458)	M1 A1 **[2]**	Allow errors. No need to see (0 × 5)
2(a)	State or imply $\int \frac{1}{N}\,\mathrm{d}N = \int -0.3\mathrm{d}t$ State ln $N = -\,0.3t$ (+ constant) Substitute N = 50 000 and t = 0 to find constant Obtain constant = ln 50 000 and use log law to obtain $\ln\frac{N}{50000} = -0.3t$ Obtain N = $50\,000\mathrm{e}^{-0.3t}$	B1 B1 M1 DM1 A1 **[5]**	Alternative If candidate recognize this equation as exponential growth and decay $\frac{dN}{dt} = kN$ $k = -0{,}3$ $N = N_0 e^{kt}$ $N = N_0$ $N = 50\ 000e^{-0{,}3t}$
2(b)	Obtain answer rounding to 15 000 (15059.7)	B1 **[1]**	
2(c)		B1 B1 **[2]**	Shows decreasing exponential shape. Axes labelled and (0, 50 000) indicated.

3(a)	Use $\mathbf{F}\times\mathbf{r}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 0 & 2\\ 1 & 2 & -1\end{vmatrix}$ E.g. $\mathbf{i}\begin{vmatrix}0 & 2\\ 2 & -1\end{vmatrix}-\mathbf{j}\begin{vmatrix}1 & 2\\ 1 & -1\end{vmatrix}+\mathbf{k}\begin{vmatrix}1 & 0\\ 1 & 2\end{vmatrix}$ Obtain $-4\mathbf{i}+3\mathbf{j}+2\mathbf{k}$	M1 A1 A1 **[3]**	Award for first step in a correct method for evaluating the cross product Award for intermediate step seen Accept equivalent forms
3(b)	Find length of their $\mathbf{F}\times\mathbf{r}$ Obtain $\sqrt{29}$	M1 A1 **[2]**	Allow any correct method Allow 5.39
3(c)	State "perpendicular to the plane" or "at 90°" or " at right angle" or equivalent	B1 **[1]**	
4(a)	State or imply a binomial distribution with $p = 0.2$ Use ${}^{6}C_{2}(0.2)^{2}(0.8)^{4}$ or $C_{2}^{6}(0.2)^{2}(0.8)^{4}$ Obtain 0.246 (0.24576)	M1 A1 A1 **[3]**	 Solution using table is accepted.
4(b)	State or imply $P(X \geq 1) = 1 - P(X = 0)$ Obtain $1-(0.8)^{n} > 0.95$ Use log law correctly : $n\log 0.8 < \log 0.05$ Show correct inequality signs : $n\log 0.8 < \log 0.05$ and $n > \dfrac{\log 0.05}{\log 0.8}$ (13.425) Obtain 14	M1 A1 A1 A1 B1 **[5]**	 Trial and error leading to 14, justifying answer from $1-(0.8)^{n}$ using $n = 13$ (0.945) and 14(0.956). Award 5/5. Obtain 14 without justification:

			award M1A1 B1
5	Equate the two **r**'s and form simultaneous equations Obtain at least two of $16 + 3\lambda = -3 + 5\mu$ $2 + 2\lambda = 8 - 6\mu$ $3 - \lambda = 12 - 3\mu$ Solve two of the equations simultaneously Obtain $\lambda = -3$ or $\mu = 2$ Obtain $7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$	M1 A1 M1 A1 A1 **[5]**	
6(a)	Obtain $\frac{dx}{dt} = -10A\sin 10t + 10B\cos 10t$ Obtain $\frac{d^2x}{dt^2} = -100A\cos 10t - 100B\sin 10t$ Substitute in $\frac{d^2x}{dt^2}$ and $100x$ to obtain 0	B1 B1 B1 **[3]**	ALTERNATIVE: Obtain equation of the form $\lambda^2 + 100 = 0$ Obtain $\lambda = \pm 10i$ Write $x = A\cos 10t + B\sin 10t$
6(b)	Substitute $\frac{\pi}{10}$ in x and obtain $A = 2$ Substitute $\frac{\pi}{10}$ in $\frac{dx}{dt}$ and obtain $B = -1$. State $x = 2\cos 10t - \sin 10t$	B1 B1 B1 **[3]**	

6(c)	Expand $r\cos(10t+\alpha)$ and compare coefficients Obtain $r\sin\alpha = 1$ AND $r\cos\alpha = 2$ State $r = \sqrt{5}$ ONLY Solve $\tan\alpha = 0.5$ or 2 Obtain $\alpha = 0.464$ rad (Allow 26.6°) State $x = \sqrt{5}\cos(10t + 0.464)$	M1 A1 B1 M1 A1 A1 **[6]**	 Accepted if the angle is located on sine or cosine
7(a)	Show use of $z = \dfrac{x-\mu}{\sigma}$ Obtain $z = 0.6$ Obtain 0.726	M1 A1 A1 **[3]**	
7(b)	Use $P(Z \le a) = 0.95$ Obtain $z_1 = 1.645$ Use $1.645 = \dfrac{x-80}{5}$ and obtain 88.2 (minutes) Obtain $P(Z \le b) = 0.75$ and use $0.674 = \dfrac{x-80}{5}$ Obtain 83.37 and hence 5 (minutes).	M1 A1 A1 M1 A1 **[5]**	

8(a)	Either substitute $T = 2.5$ into the given equation OR rearrange as $0.02T^3 + 0.11T^2 - 1 = 0$ and substitute $T = 2.5$ Show evaluation to 0.16 on both sides OR to 0	M1 A1 **[2]**	
8(b)	Sketch $y = \frac{1}{T^2}$ with asymptotes $T = 0$ and $y = 0$ clearly shown Sketch $y = 0.11 + 0.02T$. Look for linear graph with small upward gradient and intersecting the y-axis just above the origin. Show one intersection only in first quadrant with axes labelled y and T.  State $T = 0$ and $y = 0$.	B1 B1 dep B1 B1 **[4]**	 B1 dependent on both previous B marks. B0 if graphs not drawn on the same axes
8(c)	State one intersection implies one root or similar	dep B1 **[1]**	B0 if graphs not drawn on the same axes.
9(a)	State or imply $V = 6x^2h$ or $S = 10xh + 6x^2$ Substitute an expression for h in $S = 10xh + 6x^2$ Obtain with no errors seen $S = \frac{1500}{x} + 6x^2$	B1 M1 A1 **[3]**	 Given answer

9(b)	Obtain $\frac{\mathrm{d}S}{\mathrm{d}x}=\frac{-1500}{x^2}+12x$	B1	
	Equate $\frac{\mathrm{d}S}{\mathrm{d}x}$ to 0 and make a first step in solving	M1	
	Obtain x = 5 and h = 6	A1	
	State dimensions 10,15 and 6 (metres)	A1	
	Obtain $\frac{d^2S}{dx^2}=\frac{3000}{x^3}+12$ (allow one error)	M1	ALTERNATIVES: Obtain at least one value of the first derivative OR obtain two y values either side of x = 5.
	Obtain $\frac{d^2S}{dx^2}=36$ or provide an argument that $\frac{3000}{x^3}+12$ must be a positive quantity.	A1	Obtain correct value(s) of the first derivative on at least one side of x = 5 or provide correct y values on either side.
	State $\frac{d^2S}{dx^2}$ positive hence minimum	A1 **[7]**	

10(a)	Use P(X =2) = $\frac{2^2}{2!}e^{-2}$ Obtain 0.271 (0.27067)	M1 A1 **[2]**	
10(b)	Use 1 – (P(X =0) + P(X = 1)) Obtain 0.594 (0.59399)	M1 A1 **[2]**	
10(c)	P(Y = 2) = $\frac{3^2}{2!}e^{-3}$ seen Use their product of P(X =2) and P(Y = 2) Obtain 0.061	B1 M1 A1 **[3]**	 Accept $9e^{-5}$
10(d)	Show that 3 cases are considered : P(X = 0 and Y = 0), P(X = 1 and Y = 0) and P(X = 0 and Y = 1) Show the addition of 3 products Obtain $(e^{-2})(e^{-3}) + (e^{-2})(3e^{-3}) + (2e^{-2})(e^{-3})$ Obtain $6e^{-5}$ or 0.0404	B1 M1 A1 A1 **[4]**	Alternative $\lambda = 2+3=5$ $P(X<2)$ or $P(0)+P(1)$, or $P(X \le 1)$ 0,0404 (from the table) 0,040

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